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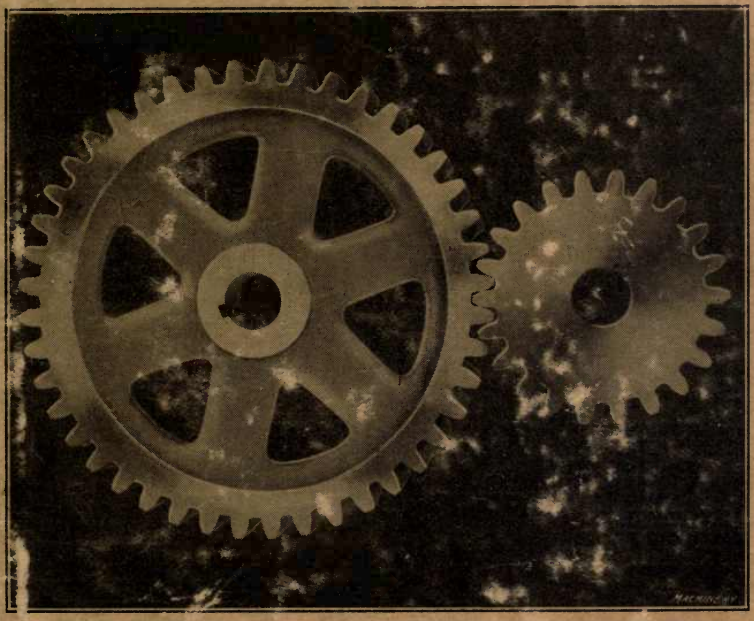
SPUR GEARING

RULES AND FORMULAS—INTERNAL SPUR GEARS—DESIGN OF SPUR GEARS

THIRD REVISED EDITION



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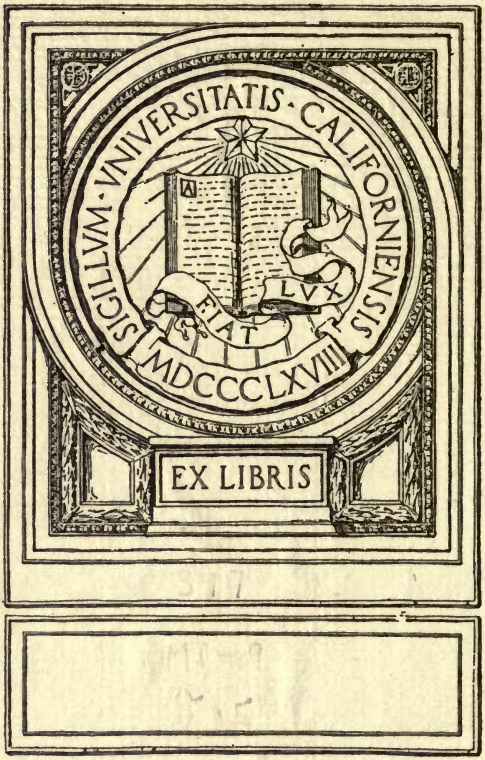
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NUMBER 15

SPUR GEARING

THIRD REVISED EDITION

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CHAPTER I

FIRST PRINCIPLES OF GEARING*

Gear wheels are such common objects about the machine shop, and are manufactured with such rapidity and ease by the aid of the modern automatic gear cutter, that many seldom stop to think what they really are, why the teeth must be constructed with certain curves, and what it is desired that they shall accomplish. In following chapters we shall take up some of the practical questions, touching upon the calculations that come up in the design, but will here deal chiefly with a few of the theoretical points of the subject that are seldom explained in a simple manner for the benefit of those who have had neither the time nor the opportunity to look into matters of this kind.

Suppose there are two wheels arranged as in Fig. 1 with their faces in close, frictional contact, and that both are exactly the same size, so

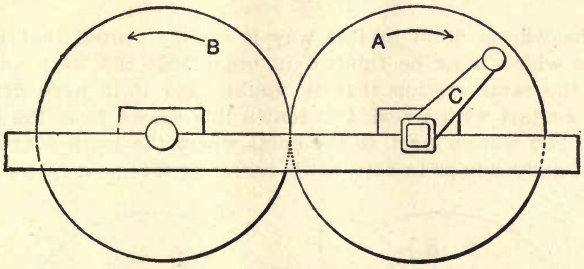

Fig. 1

that when the crank is turned around once, wheel *B* will turn exactly once also, provided, of course, there is no slipping between the two wheels. It must be noticed, moreover, that if the crank be turned uniformly, wheel *B* will not only make the correct number of revolutions relative to *A*, but it will revolve uniformly, as well; that is, both its total motion and the motion from point to point will be correct.

Now there are many places in machine construction where the slipping inseparable from friction wneels cannot be tolerated, and this difficulty might be overcome by fastening small projections to one of the wheels, as on *A* in Fig. 2, and cutting grooves in the other wheel, *B*. Then, if the crank were turned, wheel *B* would always make just the right number of turns, even if considerable power were transmitted. It is probable, however, that these projections and grooves would not fulfill the purpose of gear teeth. What is wanted of gear teeth is that they shall give exactly the same kind of motion as corresponding friction wheels, running without slipping. They must not only keep

* MACHINERY, June, 1898.

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the number of revolutions right, but they must give a perfectly even and smooth motion from point to point or from tooth to tooth.

Fig. 3 will show clearly how such a result is obtained. It represents the friction wheels with teeth fastened to them, the teeth, of course, extending all the way around instead of part way as shown. These teeth are set so as to be partly without and partly within the edges of the two wheels, as obviously they will give better results thus arranged than with all the projections on one wheel and all the grooves or depressions on the other, as in Fig. 2.

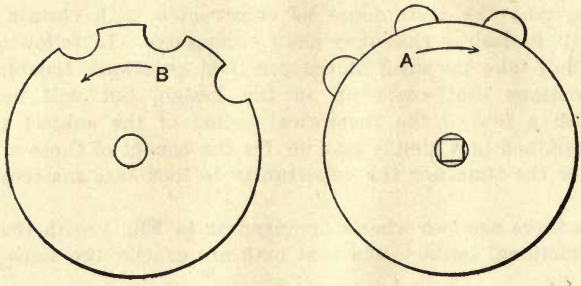


Fig. 2

With the wheels fitted in this way it can be proved that the only conditions which must be fulfilled in order that the teeth shall give wheel *B* the same motion that it would have if it were driven by frictional contact with wheel *A* is that a line drawn from the point *O*, where the two wheels meet, to the point where the tooth curves touch, shall be at right angles to both tooth curves at this point, whatever the

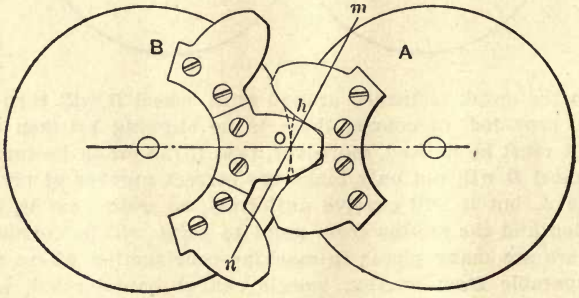


Fig. 3

position of the gears. For example, in Fig. 3, two of the teeth touch at *h*. If the curves are of the right shape, a line *m n*, drawn through *h* and *O*, will be at right angles to both curves at point *h*. This is the law of tooth curves, and it makes no difference what the shape of the teeth is, so far as their correct action is concerned, if this law holds true for every successive point where the teeth come in contact.

In technical language the "friction wheels" mentioned are known as "pitch cylinders," and they are always represented on a gear drawing by a line—usually a dash and dot line—called the "pitch line." As

teeth are generally proportioned, this line falls nearly, but not quite, midway between the tops and bottoms of the teeth, the inequality being due to the space left at the bottom of the teeth for clearance. The diameter of the pitch cylinder is called the "pitch diameter."

Involute System

We are now ready to consider the particular forms of teeth most often used. The one that is at present most in favor is the involute tooth, the term "involute" being the name of a curve described by the

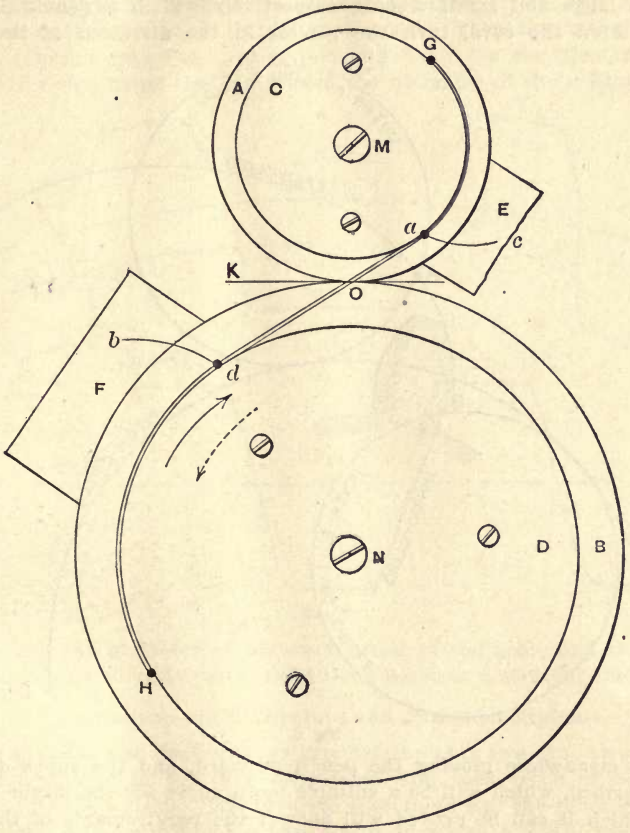


Fig. 4

end of a cord as it is unwound from another curve. For example, to draw an involute, wind a cord around a circular disk of any convenient material, and make a loop in the outer end of the cord. Lay the disk flat on a piece of paper, and with a pencil in the loop, unwind the string, keeping it drawn tight, and let the point of the pencil trace a curve, which will then be an involute.

In Fig. 4 is shown how the same principle is applied to forming tooth curves. *A* and *B*, with centers at *M* and *N*, are two disks which

serve the purpose of pitch cylinders. *C* and *D* are two smaller disks fastened to the larger ones and around which a cord is stretched and fastened at points *G* and *H*. When either disk is turned, the cord is supposed to pull the other one around at the same speed that it would go if moved solely by frictional contact between disks *A* and *B*. To do this, it is simply necessary to have the disks *C* and *D* in the same ratio as *A* and *B*. If *A*, for example, is half as large as *B*, then *C* must be half as large as *D*.

To make room for drawing the curves, let pieces *F* and *E* be fastened to the large and small wheels, respectively. With a pencil fixed at point *d* on the cord, turn the wheels in the direction of the solid

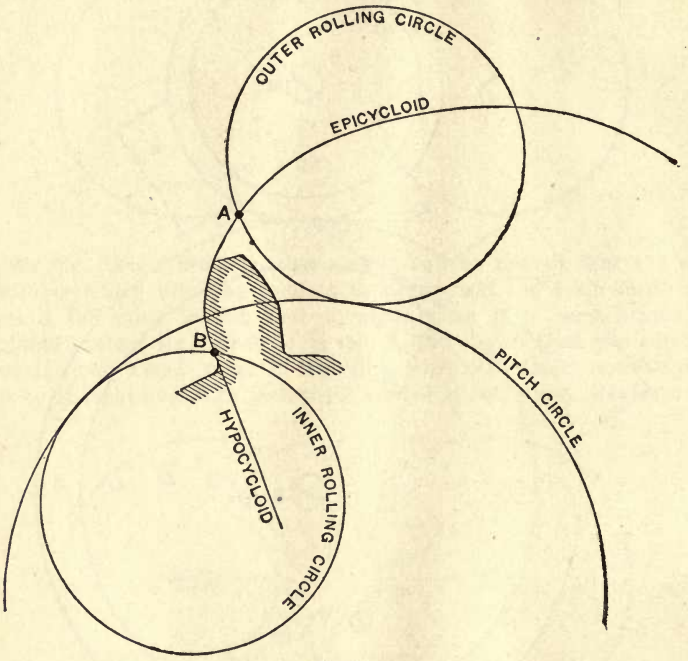


Fig. 5

arrow, meanwhile moving the pencil outward, and the curve *db* will be described, which will be a suitable tooth curve for the larger wheel, and which it can be proved will answer the requirements of the general law. Starting again with the pencil at *a*, and turning the wheels in the direction of the dotted arrow, and moving the pencil outward, a similar curve, *ac*, for the smaller wheel will be traced.

The circles representing the disks *C* and *D* are called "base circles," and in practice are drawn at a distance from the pitch circle of about one-sixtieth of the pitch diameter. This brings the angle, *KOd*, called the angle of obliquity, in Fig. 4, about 14½ degrees; and although it is not by any means certain that this is the best angle, it is the one commonly used.

Cycloidal System

Take a silver dollar and roll it along the edge of a ruler, holding the point of a pencil at the rim of the dollar, so that as the latter rolls, the pencil will trace a curve. This curve is a cycloid. Should the dollar be rolled on the edge of a circular disk, however, the curve traced would be an epi-cycloid, and should it be rolled on the inside of a hoop, it would be called a hypo-cycloid. These curves are employed for the teeth of the cycloidal system of gears.

In Fig. 5 it is shown how the face or the outer portion of the tooth is rolled up by the point *A* on the outer rolling circle, and how the flank or inner portion is generated by point *B* on the inner rolling circle. In this case the hypo-cycloid and flank are straight lines, the reason for this being that, as drawn, the diameter of the rolling circle

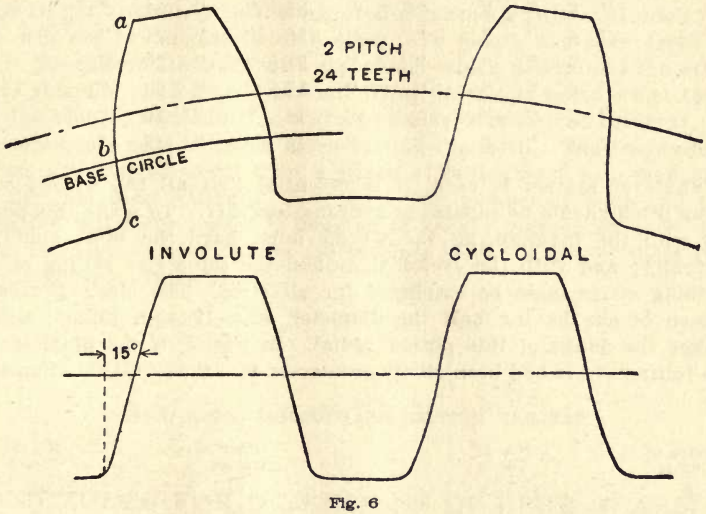


Fig. 6

is one-half the diameter of the pitch circle of the gear, and the hypo-cycloid generated under these conditions becomes a straight line.

Comparison of the Involute and Cycloidal Systems

The involute and cycloidal systems are the only two that are used to any extent, and in Fig. 6 a gear tooth and rack tooth of both are shown for comparison. The involute gear tooth has the involute curve from point *a* to point *b* on the base circle, and from *b* to *c* at the bottom of the tooth the flank is a straight, radial line. One difficulty with the involute system is that with the standard length of tooth the point *a* will interfere when running with gears or pinions having a small number of teeth. To avoid this, the point is rounded off a little below the involute curve. In general appearance the tooth seems to have a broad, strong base, and a continuous curve from *a* to *c*. A strong feature of the involute gearing is that it will run correctly even if the distance between the centers of the wheels is not exactly right. This

will be evident by referring to Fig. 4, where it will appear that the relative velocities of the two wheels will be the same however far apart they may be, and if involute teeth are used in place of the string connection there shown, the action will be just the same. The involute rack tooth has straight sides at an angle of 14½ degrees, with the points rounded off.

Of the cycloidal teeth but little need be said except that they have two distinct curves above and below the pitch line, as previously explained, and that in the rack tooth the two curves are just alike, but reversed.

TABLE I. CUTTERS FOR INVOLUTE GEAR TEETH

No.	1	will	cut	wheels	from	135	teeth	to	a rack.	
"	2	"	"	"	"	55	"	"	134	teeth.
"	3	"	"	"	"	35	"	"	54	"
"	4	"	"	"	"	26	"	"	34	"
"	5	"	"	"	"	21	"	"	25	"
"	6	"	"	"	"	17	"	"	20	"
"	7	"	"	"	"	14	"	"	16	"
"	8	"	"	"	"	12	"	"	13	"

Whatever system is used, it is essential that all the wheels of a given pitch should be capable of running together. To make this possible with the involute, all the wheels must have the same angle of obliquity; and with the cycloidal system the same size rolling or describing circle must be employed for all sizes. The circle generally chosen is one having half the diameter of a 12-tooth pinion, which makes the flanks of this pinion radial. In Fig. 5, if the diameter of the rolling circle had been either greater or less than half the diameter

TABLE II. CUTTERS FOR CYCLOIDAL GEAR TEETH

Letter of Cutter	No. of Teeth	Letter of Cutter	No. of Teeth
A	12	M	27 to 29
B	13	N	30 to 33
C	14	O	34 to 37
D	15	P	38 to 42
E	16	Q	43 to 49
F	17	R	50 to 59
G	18	S	60 to 74
H	19	T	75 to 99
I	20	U	100 to 149
J	21 to 22	V	150 to 249
K	23 to 24	W	250 or more
L	25 to 26	X	Rack

of the pitch circle, the flank of the tooth would have been curved, and in the case of the greater circle, the curve would have fallen inside of the radial flank drawn in the figure, causing a weak, under-cut tooth. With the smaller circle, the curve would fall outside, making a strong tooth.

Cutters for Involute and Cycloidal Teeth

According to the system for cutting gear teeth adopted by the Brown & Sharpe Mfg. Co., Providence, R. I., any gear of one pitch will mesh with any other gear or with a rack of the same pitch. Eight cutters are required for each pitch. These eight cutters are adapted to cut from a pinion of twelve teeth to a rack, and are numbered, respectively, 1, 2, 3, etc. The number of teeth and the pitch for which a cutter is adapted is always marked on each. A list of these cutters is given in Table I.

Cutters for the cycloidal form of teeth are also made so that any gear of one pitch will mesh into any other gear or into a rack of the same pitch, but twenty-four cutters are required for each pitch. In order that gears with this form of teeth shall run well together, they must be cut accurately to the required depth; otherwise the pitch circles will not be tangent to each other. To secure a proper depth of tooth, the cutters are made with a shoulder which determines the exact depth that the tooth should be cut. Thus, if care is taken when turning the blanks, to obtain the correct outside diameter of the gear, no measurements need be taken when cutting the teeth. The twenty-four cutters are adapted to cut from a pinion of twelve teeth to a rack, and are designated by letters *A*, *B*, *C*, etc. The number of teeth and the pitch for which the cutter is adapted is always marked on each, the same as in the case of cutters for involute teeth. A list of these cutters is given in Table II.

CHAPTER II

FORMULAS FOR DIMENSIONS OF SPUR GEARS*

When we consider the number of gears used in machinery, and the number of men employed in the manufacture of machines using gears, it is rather surprising to find men who are unable to find the outside diameter, having given the pitch diameter and pitch, or to find the distance between centers of two gears, having given the number of teeth and pitch, and similar problems. The object of this chapter is to explain in as clear and practical a manner as possible the underlying principles of gearing, and to give concise rules or formulas for the solution of problems which arise in our everyday work upon gears.

Pitch Diameters

Two shafts *A* and *A'* (Fig. 7) carry rollers *B* and *B'*. By having pressure on the shafts as indicated by the arrows, and revolving *A*, the friction of the rollers at the point of contact, *X*, will cause *A'* to revolve, but we can readily see that if any great amount of power is to be

* MACHINERY, July, August, October and November, 1897.

transmitted, the rollers are liable to slip at the point of contact *X*, which will not give a positive motion; that is, it will require more than one revolution of the shaft *A* to produce one revolution of the shaft *A'*.

Suppose, as shown in Fig. 8, that we put projections on the surface of the roller *B* and cut recesses in the roller *B'*, making them of such shape that the sides of the projections on roller *B* will slide with as little friction as possible upon the sides of the projections caused by cutting the recesses in roller *B'*. Then, when shaft *A* is revolved, shaft

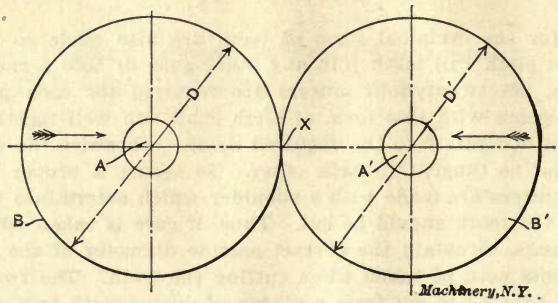


Fig. 7

A' must also revolve. The identity of the rollers *B* and *B'* is not lost, for we have simply added a number of projections to one, and cut the same number of recesses in the other, and the point of contact of the two rollers is still at *X*, but in this case there is no special pressure required to keep the rollers together as in the preceding case, nor is

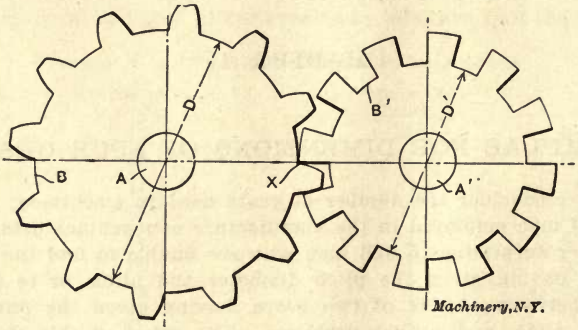


Fig. 8

there any slip, and consequently shaft *A'* will make one revolution in the same time that shaft *A* does.

In Fig. 9 we have changed Fig. 8 by adding projections between recesses in roller *B'*, and by cutting recesses on roller *B* between projections, and we have the regular gear tooth. We have now no visible part of the original rollers *B* and *B'* left, but we have in their places imaginary rollers, the diameters of which are the pitch diameters of the gears. Thus we might have called our original rollers pitch rollers, and then proceeded to put on our projections and cut our recesses,

which would have given us the gear wheel. This has already been explained in a general way in Chapter I.

Of course, in practice gears are never made in this way; the gear blank is first turned up to the correct diameter, and then the space between the teeth is cut. The method of finding the outside diameter will be given later, this illustration being used simply to show the evolution of the gear wheel from the friction disks or pitch rollers.

Pitch

When we speak of the pitch of a gear, the diametral pitch is generally referred to. The gear really has two pitches, diametral and circular. The *diametral pitch* of a gear is the number of teeth for each inch of pitch diameter. If a gear has 20 teeth and the pitch diameter is 2 inches, the diametral pitch would equal $20 \div 2$, or 10; or there are 10 teeth in the gear for each inch of pitch diameter which it contains, and we would call it a 10-pitch gear. The *circular pitch*

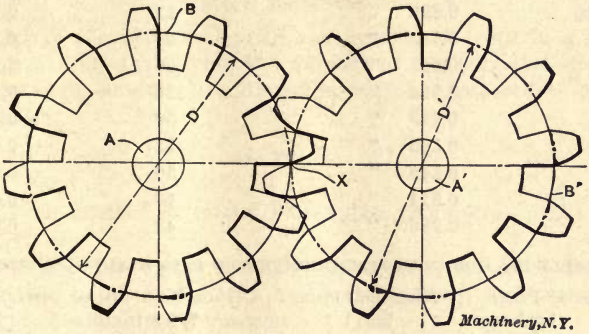


Fig. 9

of a gear is the distance from the center of one tooth to the center of the next adjacent tooth, measured on the pitch lines. It is very seldom that circular pitch is used in describing cut gears.

It can readily be seen that the circular pitch being equal to the distance from the center of one tooth to the center of the next, must be the result of dividing the circumference of the pitch circle by the number of teeth in the gear. Should an occasion arise where it would be necessary to obtain the circular pitch, having the diametral pitch given, divide 3.1416 by the diametral pitch, and the quotient will be the circular pitch, or, expressed in its simplest form,

$$\frac{3.1416}{P} = P' \qquad (1)$$

in which P = diametral pitch; P' = circular pitch.

Example.—If the diametral pitch of a gear is 4, and it is required to find the circular pitch, divide 3.1416 by 4, and the quotient, 0.7854, is the circular pitch of the gear.

If the circular pitch be given, to find the diametral pitch, we can readily see that formula (1) would have to be transposed and would read thus:

$$\frac{3.1416}{P'} = P \qquad (2)$$

P and P' representing the same as before.

Now, having given the rules, we will proceed to explain how they were obtained. We know that the distance around the circumference of a circle is equal to 3.1416, multiplied by the diameter of the circle; consequently, for every inch of diameter we have 3.1416 inches of cir-

TABLE III. DIAMETRAL PITCH CONVERTED INTO CIRCULAR PITCH

Diametral Pitch	Circular Pitch	Diametral Pitch	Circular Pitch
2	1.571 inch.	12	0.262 inch.
2¼	1.396 "	14	0.224 "
2½	1.257 "	16	0.196 "
2¾	1.142 "	18	0.175 "
3	1.047 "	20	0.157 "
3½	0.898 "	22	0.143 "
4	0.785 "	24	0.131 "
5	0.628 "	26	0.121 "
6	0.524 "	28	0.112 "
7	0.449 "	30	0.105 "
8	0.393 "	32	0.098 "
9	0.349 "	36	0.087 "
10	0.314 "	40	0.079 "
11	0.286 "	48	0.065 "

TABLE IV. CIRCULAR PITCH CONVERTED INTO DIAMETRAL PITCH

Circular Pitch	Diametral Pitch	Circular Pitch	Diametral Pitch
2 inches.	1.571	7/8 inch.	3.590
1 7/8 "	1.676	13/16 "	3.867
1 3/4 "	1.795	3/4 "	4.189
1 5/8 "	1.933	11/16 "	4.570
1 1/2 "	2.094	5/8 "	5.027
1 7/16 "	2.185	9/16 "	5.585
1 3/8 "	2.285	1/2 "	6.283
1 5/16 "	2.394	7/16 "	7.181
1 1/4 "	2.513	3/8 "	8.378
1 3/16 "	2.646	5/16 "	10.053
1 1/8 "	2.793	1/4 "	12.566
1 1/16 "	2.957	3/16 "	16.755
1 "	3.142	1/8 "	25.133
15/16 "	3.351	1/16 "	50.266

cumference. If the diametral pitch of a gear is equal to the number of teeth for each inch of pitch diameter, and each inch of diameter is represented by 3.1416 inches of circumference, then the diametral pitch equals number of teeth for each 3.1416 inches of circumference. As the circular pitch is the distance from the center of one tooth to the center of the next, then the circular pitch must be equal to 3.1416 divided by the number of teeth in that 3.1416 inches of circumference, and, as we have shown that the diametral pitch is equal to the number of

teeth in each 3.1416 inches of circumference, then the circular pitch must equal 3.1416 divided by the diametral pitch, which proves formula (1).

It may not be actually necessary to show how we obtain the diametral pitch from the circular pitch, but we will endeavor to explain everything as we go along. As in the preceding case, we begin with the ratio of the circumference of the circle to its diameter, which is 3.1416. In each 3.1416 inches of circumference we have a certain number of teeth, which is the diametral pitch of the gear. Now, having given the circular pitch, if we divide 3.1416 by that, we obtain the number of teeth for 3.1416 inches of the circumference, which is the diametral pitch of the gear, which proves formula (2).

The accompanying tables will facilitate the finding of corresponding diametral and circular pitches. Table III gives the even diametral pitches with the corresponding circular pitches, while Table IV gives the even circular pitches with the corresponding diametral pitches.

Pitch Diameter

Having given the diametral pitch and number of teeth in a gear, to find the pitch diameter, divide the number of teeth by the pitch, and the quotient will be the pitch diameter, which, expressed in its simplest form, is:

$$\frac{N}{P} = D \qquad (3)$$

in which N = number of teeth; P = pitch (diametral); D = pitch diameter.

Example.—A 10-pitch gear has 35 teeth, what is the pitch diameter? Divide 35 (the number of teeth) by 10 (the pitch), and the quotient 3½ is the pitch diameter of the gear.

The definition of diametral pitch proves this formula. If the diametral pitch equals the number of teeth to each inch of pitch diameter, then dividing the number of teeth in the gear by the diametral pitch will give the number of inches of the pitch diameter. If the circular pitch and number of teeth are given, first find the diametral pitch, and proceed as given above.

Addendum

The addendum of a gear tooth is the distance from the pitch circle to the outside circumference of the gear. This distance is always equal to the reciprocal of the diametral pitch, or 1 divided by the diametral pitch, and, expressed as a formula, is:

$$S = \frac{1}{P} \qquad (4)$$

in which S = addendum; P = diametral pitch.

Outside Diameter

When we start to make a gear, we first wish to know the outside diameter. If we have the pitch and number of teeth given, this may easily be found by the following rule: Add 2 to the number of teeth,

and divide by the pitch. This, expressed as a formula, is:

$$\frac{N+2}{P}=O \qquad (5)$$

in which N = number of teeth; P = diametral pitch; O = outside diameter.

Example.—Given a gear of 20 teeth and 4 pitch, to find the outside diameter. The number of teeth, 20, plus 2 equals 22, and 22 divided by 4 (the pitch of the gear) equals 5½, the outside diameter of the gear.

This formula is simply a combination of formulas (3) and (4), for we first find the pitch diameter, and then add the addendum twice, for it must be added on each side of the pitch diameter. The mathematical solution is as follows:

$$\frac{N}{P}=D;\quad D+\frac{1}{P}+\frac{1}{P}=O$$

$$O=D+\frac{2}{P};\quad O=\frac{N+2}{P} \qquad (5)$$

Dedendum and Clearance

The dedendum is the working depth of the tooth below the pitch line, and must be equal to the addendum or $\frac{1}{P}$, for the pitch circles of two gears are tangent (touching), so the addendum of one will give the working depth of the other below the pitch line. The clearance is the distance from the end of the dedendum to the bottom of the space between the teeth. There is no common standard for this distance, different gear makers using different distances, yet the difference between them is very slight.

The Brown & Sharpe formula for this distance is:

$$F=\frac{0.157}{P} \qquad (6)$$

in which F = clearance; P = diametral pitch.

The Geo. B. Grant formula is:

$$F=\frac{S}{8} \qquad (7)$$

in which F = clearance; S = addendum.

Thickness of Tooth

The thickness of tooth and width of the space of a gear are always equal at the pitch line, and if the circular pitch is the distance from the center of one tooth to the center of the next tooth measured on the pitch line, tooth and space being equal, then the thickness of tooth must be equal to one-half the circular pitch, or

$$T=\frac{P'}{2} \qquad (8)$$

in which T = thickness of tooth at pitch line; P' = circular pitch.

We know by formula (1) that

$$P' = \frac{3.1416}{P} \qquad (1)$$

and substituting this value for P' in formula (8) we have:

$$T = \frac{\frac{3.1416}{P}}{2}$$

and this formula resolved to its simplest form is:

$$T = \frac{1.5708}{P} \qquad (9)$$

in which T = thickness of tooth at pitch line; P = diametral pitch.

Example.—Given a gear 1 3/16 circular pitch, what is the thickness of tooth at the pitch line? 1 3/16 (the circular pitch) divided by 2 gives 19/32, the thickness of tooth at the pitch line.

Example.—Given a 6-pitch gear to find the thickness of tooth at the pitch line. 1.5708 divided by 6 (the diametral pitch of the gear) gives 0.262, the thickness of tooth at the pitch line.

Table V gives the thickness of tooth at the pitch line for the different diametral pitches.

Depth of Tooth

After we get the gear blank turned up, we next want to know how deep to run the gear cutter in order to get a perfect tooth. The working depth of the tooth we have shown to be equal to the sum of the addendum and dedendum, or $\frac{1}{P} + \frac{1}{P} = \frac{2}{P}$, and the whole depth of the tooth must equal $\frac{2}{P}$ plus the clearance.

Using the Brown & Sharpe standard, we have $\frac{2}{P} + \frac{0.157}{P} =$

$$W = \frac{2.157}{P} \qquad (10)$$

in which 1 = whole depth of tooth; P = diametral pitch.

Example.—Given a gear of 6 diametral pitch, to find the depth of cut to be taken to get a perfect gear tooth.

Divide 2.157 by 6 (diametral pitch) and the quotient 0.359 is the depth to be cut in the gear.

If we had the circular pitch given, to find the depth of tooth, we could substitute in formula (10) the value of P as given in the formula (2), and we would have

$$W = \frac{2.157}{3.1416 \div P'}$$

which, reduced to its simplest form, is:

$$W = 0.6866\ P' \qquad (11)$$

in which W = depth to be cut in gear;

P' = circular pitch.

Example.—Given a gear 1½ inch circular pitch, to find the depth to be cut.

Multiply 0.6866 by 1½ (circular pitch), and the product 1.030 is the depth to be cut in gear.

Table VI gives the depth to be cut in a gear for different diametral pitches.

TABLE V. THICKNESS OF TOOTH AT PITCH LINE

Diametral Pitch	Thickness of Tooth at Pitch Line	Diametral Pitch	Thickness of Tooth at Pitch Line
2	0.785 inch.	12	0.131 inch.
2¼	0.697 "	14	0.112 "
2½	0.628 "	16	0.098 "
2¾	0.570 "	18	0.087 "
3	0.523 "	20	0.079 "
3½	0.448 "	22	0.071 "
4	0.393 "	24	0.065 "
5	0.314 "	26	0.060 "
6	0.262 "	28	0.056 "
7	0.224 "	30	0.052 "
8	0.196 "	32	0.049 "
9	0.175 "	36	0.044 "
10	0.157 "	40	0.039 "
11	0.143 "	48	0.033 "

Distance Between Centers

Having given the number of teeth and diametral pitch of two gears, to find the distance between centers, add the number of teeth together, and divide by twice the diametral pitch, or

$$\frac{N_g + N_p}{2P} = C \qquad (12)$$

in which N_p = number of teeth in one gear,

N_g = number of teeth in other gear,

P = diametral pitch,

C = distance between centers.

This formula is obtained from formula (3):

$$\frac{N}{P} = D$$

This formula gives us the pitch diameter of one gear, and, if we get the pitch diameters of two gears and add them together, we have twice the distance between centers, for the sum of the pitch diameters is twice the sum of the pitch radii, which is the distance between centers.

We have now traced, by the aid of a few "rules," the proportions of a gear tooth, having given the pitch and number of teeth, through

pitch diameter, addendum, dedendum, clearance, width of tooth and depth to be cut, up to the distance between centers. We now give some formulas for the solution of problems in which some of the quantities which were known in preceding problems are unknown.

Pitch

1. To find the pitch, having given the pitch diameter and number of teeth. Divide the number of teeth by the pitch diameter, and the

TABLE VI. DEPTH OF TOOTH

Diametral Pitch	Depth to be cut in gear	Diametral Pitch	Depth to be cut in gear
2	1.078 inch.	12	0.180 inch.
2¼	0.958 "	14	0.154 "
2½	0.863 "	16	0.135 "
2¾	0.784 "	18	0.120 "
3	0.719 "	20	0.108 "
3½	0.616 "	22	0.098 "
4	0.539 "	24	0.090 "
5	0.431 "	26	0.083 "
6	0.359 "	28	0.077 "
7	0.308 "	30	0.072 "
8	0.270 "	32	0.067 "
9	0.240 "	36	0.060 "
10	0.216 "	40	0.054 "
11	0.196 "	48	0.045 "

quotient will be the pitch. The proof of this assertion is derived from the formula:

$$D = \frac{N}{P} \qquad (3)$$

If the pitch diameter equals the number of teeth divided by the pitch, then the pitch diameter multiplied by the pitch must equal the number of teeth; therefore the pitch must equal the number of teeth divided by the pitch diameter, and this, expressed in its simplest form, is:

$$P = \frac{N}{D} \qquad (13)$$

in which $P =$ pitch (diametral); $N =$ number of teeth in gear; $D =$ pitch diameter.

Example.—A gear, 3 inches pitch diameter, has 36 teeth. Find the diametral pitch.

Divide 36 (the number of teeth) by 3 (the pitch diameter), and we have 12, the diametral pitch of the gear.

2. Having given the outside diameter and number of teeth, to find the diametral pitch. Add 2 to the number of teeth, and divide by the outside diameter, and the quotient will be the pitch of the gear.

In formula (5) we have:

$$\frac{N+2}{P} = O \qquad (5)$$

If the number of teeth + 2 divided by the pitch equals the outside diameter, then the outside diameter multiplied by the pitch must equal the number of teeth + 2, and then the pitch must equal the number of teeth + 2 divided by the outside diameter, which, expressed as a formula, is:

$$\frac{N+2}{O}=P \qquad (14)$$

in which N = number of teeth in gear; O = outside diameter; P = diametral pitch.

Example.—Given a gear of 36 teeth and 3 1/6-inch outside diameter; to find the diametral pitch.

36 (the number of teeth) + 2 = 38.

38 ÷ 3 1/6 = 12, the diametral pitch of the gear.

Pitch Diameter

1. Having given the outside diameter and the pitch, to find the pitch diameter. The distance from the pitch diameter to the outside diameter is $\frac{1}{P}$, as explained in formula

$$S=\frac{1}{P} \qquad (4)$$

and as this is to be added on each side of the center, the outside diameter of the gear must be equal to the pitch diameter plus $\frac{2}{P}$. If this is so, then $\frac{2}{P}$ subtracted from the outside diameter will give the pitch diameter, or

$$D=O-\frac{2}{P} \qquad (15)$$

in which D = pitch diameter; O = outside diameter; P = diametral pitch.

Example.—Given a gear 3 1/6 inches outside diameter and 12 pitch; To find the pitch diameter.

3 1/6 (the outside diameter) — 2/12 = 3 inches, the pitch diameter of the gear.

2. Having given the outside diameter and number of teeth, to find the pitch diameter. Multiply the outside diameter by the number of teeth, and divide by the number of teeth plus 2.

We have shown in formula (5) that the outside diameter equals the number of teeth + 2 divided by pitch, or

$$O=\frac{N+2}{P} \qquad (5)$$

and in the formula (13) that pitch equals the number of teeth divided by the pitch diameter, or

$$P = \frac{N}{D} \qquad (13)$$

Now, if the outside diameter equals the number of teeth plus 2 divided by the diametral pitch (and the diametral pitch equals the number of teeth divided by the pitch diameter), then the outside diameter must be equal to the number of teeth plus 2, divided by a fraction with the number of teeth as numerator and the pitch diameter as denominator. This is simply substituting the value of the pitch as shown in formula (13) for the pitch in formula (5), and expressed as a formula, is:

$$O = \frac{N + 2}{N \div D}$$

Multiplying both sides of the equal sign by $\frac{N}{D}$ we have

$$O \times \frac{N}{D} = N + 2, \text{ or } \frac{O \times N}{D} = N + 2,$$

and now, multiplying both sides by D, we have

$$O \times N = (N + 2) \times D$$

and dividing both sides by $N + 2$ we get

$$\frac{O \times N}{N + 2} = D, \text{ or } D = \frac{O \times N}{N + 2} \qquad (16)$$

in which D = pitch diameter; N = number of teeth; O = outside diameter.

Example.—Given a gear 3 1/6 inches outside diameter and 36 teeth. to find the pitch diameter.

3 1/6 (the outside diameter) multiplied by 36 (the number of teeth) equals 114. 36 (the number of teeth) + 2 = 38. 114 ($O \times N$) divided by 38 ($N + 2$) = 3 inches, the pitch diameter of the gear.

Number of Teeth

1. Having given the pitch diameter and pitch, to find the number of teeth. Multiply the pitch diameter by the pitch, and the product will be the number of teeth in the gear.

The diametral pitch of a gear equals the number of teeth for each inch of pitch diameter; hence, if we multiply the pitch by the number of inches of pitch diameter we will have the number of teeth in the gear, which, expressed as a formula, is:

$$N = P \times D \qquad (17)$$

in which P = diametral pitch; D = pitch diameter.

Example.—Given a gear 3 inches pitch diameter and 12 diametral pitch, to find the number of teeth. 3 (pitch diameter) multiplied by 12 (diametral pitch) = 36, the number of teeth in the gear.

2. To find the number of teeth, having given the outside diameter and pitch. Multiply the outside diameter by the pitch and subtract 2, or

$$N = (O \times P) - 2 \qquad (18)$$

CHART FOR DIMENSIONS OF SPUR GEARS

No.	To Find	Rule	Formula
1	Diametral Pitch	Divide 3.1416 by circular pitch	$P = \frac{3.1416}{P'}$
2	Circular Pitch	Divide 3.1416 by diametral pitch	$P' = \frac{3.1416}{P}$
3	Pitch Diameter	Divide number of teeth by diametral pitch	$D = \frac{N}{P}$
4	Pitch Diameter	Multiply number of teeth by circular pitch and divide the product by 3.1416	$D = \frac{NP'}{3.1416}$
5	Center Distance	Add the number of teeth in both gears and divide the sum by two times the diametral pitch	$C = \frac{N_g + N_p}{2P}$
6	Center Distance	Multiply the sum of the number of teeth in both gears by circular pitch and divide the product by 6.2832	$C = \frac{(N_g + N_p)P'}{6.2832}$
7	Addendum	Divide 1 by diametral pitch	$S = \frac{1}{P}$
8	Addendum	Divide circular pitch by 3.1416	$S = \frac{P'}{3.1416}$
9	Clearance	Divide 0.157 by diametral pitch	$F = \frac{0.157}{P}$
10	Clearance	Divide circular pitch by 20	$F = \frac{P'}{20}$
11	Whole Depth of Tooth	Divide 2.157 by diametral pitch	$W = \frac{2.157}{P}$
12	Whole Depth of Tooth	Multiply 0.6866 by circular pitch	$W = 0.6866\,P'$
13	Thickness of Tooth	Divide 1.5708 by diametral pitch	$T = \frac{1.5708}{P}$
14	Thickness of Tooth	Divide circular pitch by 2	$T = \frac{P'}{2}$
15	Outside Diameter	Add 2 to the number of teeth and divide the sum by diametral pitch	$O = \frac{N+2}{P}$
16	Outside Diameter	Multiply the sum of the number of teeth plus 2 by circular pitch and divide the product by 3.1416	$O = \frac{(N+2)P'}{3.1416}$
17	Diametral Pitch	Divide number of teeth by pitch diameter	$P = \frac{N}{D}$
18	Circular Pitch	Multiply pitch diameter by 3.1416 and divide by number of teeth	$P = \frac{3.1416\,D}{N}$
19	Pitch Diameter	Subtract two times the addendum from outside diameter	$D = O - 2S$
20	Number of Teeth	Multiply pitch diameter by diametral pitch	$N = P \times D$
21	Number of Teeth	Multiply pitch diameter by 3.1416 and divide the product by circular pitch	$N = \frac{3.1416\,D}{P'}$
22	Outside Diameter	Add two times the addendum to the pitch diameter	$O = D + 2S$
23	Length of Rack	Multiply number of teeth in rack by 3.1416 and divide by diametral pitch	$L = \frac{3.1416\,N}{P}$
24	Length of Rack	Multiply the number of teeth in the rack by circular pitch	$L = NP'$

in which N = number of teeth; O = outside diameter; P = diametral pitch.

This formula is simply the reverse of formula

$$\frac{N+2}{P}=O \qquad (5)$$

If the outside diameter equals the number of teeth + 2 divided by the pitch, which we have already proved, than the number of teeth plus 2 must equal the outside diameter multiplied by the pitch, and subtracting 2 from this result we have the number of teeth in the gear.

Example.—Given a gear 3 1/6 inches outside diameter and 12 pitch, to find the number of teeth. Multiply 3 1/6 (outside diameter) by 12 (the pitch) and we have 38, and subtracting 2 from this result we have 36, the number of teeth in the gear.

Outside Diameter

To find the outside diameter having given the pitch diameter and pitch. Divide 2 by the pitch and add to the pitch diameter, or

$$O=D+\frac{2}{P} \qquad (19)$$

in which O = outside diameter,
D = pitch diameter,
P = pitch.

The addendum of a gear is $\frac{1}{P}$ [formula (4)] and this, added on each side of the pitch diameter, gives the outside diameter.

Example.—Given a gear 3 inches pitch diameter and 12 pitch; to find the outside diameter.

3 (pitch diameter) plus 2/12 $\left(\frac{2}{P}\right)$ = 3 1/6 inches, the outside diameter of the gear.

Summary of Formulas

In the chart on page 20, the rules and formulas for the dimensions of spur gears have been grouped together, so that they may be more easily found when wanted. The same reference letters are used as have previously been employed in deducing the various formulas. It will be noticed that with the aid of the formulas given in the chart, each dimension can be calculated either from the diametral or circular pitch. The first sixteen formulas are placed in the order in which they would naturally present themselves to the designer when determining the dimensions of a pair of spur gears. Nos. 17 to 22 give additional formulas for various conditions of known and unknown factors. Formulas Nos. 23 and 24 are for racks.

CHAPTER III

INTERNAL SPUR GEARS

As indicated by its name, the internal gear is one having teeth formed on an interior pitch surface instead of an exterior one. In a word, it is an ordinary spur gear turned inside out. At the right of Fig. 10 is shown a sketch of such a gear, meshing with a spur pinion; at the left is shown a pair of spur gears having the same number and pitch of teeth as the pinion and internal gear on the right. By tracing the motion in each figure, it will be seen that internal action causes the

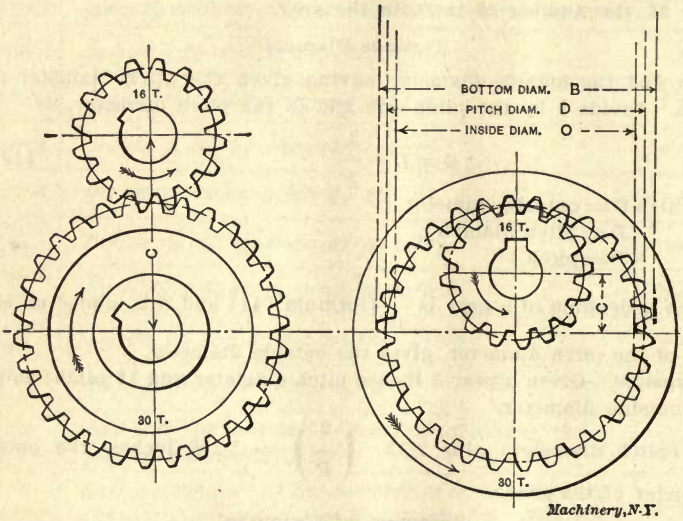


Fig. 10. Internal Spur Gears

two members to turn in the same direction, while external action produces opposite rotation.

The Uses of Internal Gearing

There are some advantages attaching to the use of internal gears for particular applications, as compared with external gears of the same pitch and number of teeth. For one thing, an internal gear has its teeth and that of its pinion protected to a very marked degree from inflicting or receiving injury, often making the use of a gear guard unnecessary if the parts are properly designed for that purpose. Owing to the fact that the cylindrical pitch surfaces in internal gearing have their curvature in the same direction, the teeth of the pinion approach and mesh with those of its mate somewhat more gradually and easily than when they are meshing with an external gear. This tends toward smoothness and quietness in running, as well as giving a slight-

ly longer contact for each tooth. Another characteristic which is often an advantage will be seen from a study of Fig. 10. In each case shown we have gears of the same pitch and number of teeth. The internal gears evidently figure out to a much smaller center distance than the external gears. This matter is of importance when it is necessary to transmit considerable power between shafts placed quite close together.

In contrast with the advantages just mentioned, the chief factor which has limited the use of the internal gear, has been the difficulty and expense of making it. This difficulty has not been insuperable for cast gearing, but, until the introduction of recent processes, the cutting of internal teeth has been tedious and unsatisfactory.

Rules for Designing Internal Gearing

Neglecting for the time being the modifications which have to be made in the standard proportions to get rid of interference, it may be said that the usual thing to do in designing internal gearing is to follow exactly the dimensions of the standard system as used for external spur gearing. Practically the only modifications required in the rules given on page 20 are those made necessary by the fact that the center distance, in internal gearing, is equal to the difference between the two pitch radii, instead of to their sum. Besides this, we have of course to reckon with the fact that the teeth are turned inside out, so that the bottom or root diameter is larger than the pitch diameter.

The only new term is "Inside Diameter," which takes the place of the outside diameter of external spur gearing. It is, of course, the inside diameter of the blank before the teeth are cut, and it is marked *O* in Fig. 10. The following are the rules which must be changed:

No. 5 will read: *To find the center distance, subtract the number of teeth in the pinion from the number of teeth in the gear and divide the remainder by* 2 *times the diametral pitch.*

No. 6 will read: *To find the center distance, multiply the difference of the numbers of teeth in the gear and pinion by the circular pitch and divide the product by* 6.2832.

No. 15 will read: *To find the inside diameter, subtract* 2 *from the number of teeth and divide the remainder by the diametral pitch.*

No. 16 will read: *To find the inside diameter, subtract* 2 *from the number of teeth, multiply the remainder by the circular pitch, and divide the product by* 3.1416.

No. 19 will read: *To find the pitch diameter, add twice the addendum to the inside diameter.*

No. 22 will read: *To find the inside diameter, subtract twice the addendum from the pitch diameter.*

Interference

In laying out the shape of teeth for internal gearing we have to look out for two kinds of interference which are almost sure to be met with. The points of the rack teeth in the 14½-degree involute system are relieved to avoid the interference with the flanks of small pinions, and the points of internal gear teeth have to be relieved for the same

reason, but to an even greater degree. A second form of interference occurs when the pinion has too nearly the same number of teeth as the gear. In this case there is a tendency for the points of the pinion and the gear teeth to strike as they roll into and out of engagement.

It would require an extended treatise to explain the different ways of avoiding these two forms of interference. For the first form correction may be made either by correcting the points of the internal gear tooth by shortening them (at the same time preferably lengthening the addendum of the pinion), or by changing the pressure angle. The mechanic who desires to use internal cut gearing without making a study of the theoretical conditions has two courses open to him. He may purchase a formed cutter for the gear from the regular makers of formed cutters, telling them the number of teeth and pitch he proposes to use for the gear and pinion. In that case, the maker of the cutter will make such corrections in its form as may be necessary to avoid interference. Another way is to cut the internal gear on the Fellows gear shaper. With this machine the cutter forms its own correction, so that no calculation on the part of the user is ordinarily required.

CHAPTER IV

STRENGTH AND DURABILITY OF SPUR GEARS

The principal materials used for gears are cast iron, steel (both from bar stock and in the form of forgings or castings), brass and bronze (from stock or castings), rawhide, and fiber.

Cast iron is, perhaps, the most used material. It is one of the cheapest, and is the easiest to mold or cut to shape. It wears fairly well also. Its greatest disadvantages are its lack of resisting power to shock or impact, and the uncertainty of the quality of the casting into which it is formed. A casting is liable to various more or less serious defects, some of which may be visible from the exterior, while others are concealed; 'blow holes," "cold shuts," "scabs," etc.," are of common occurrence where the foundry work is not skillfully done. Castings from iron are so cheap, however, that those containing such defects may be discarded without hesitation as soon as they are discovered. The prime advantage of the material, aside from its cheapness, is the facility with which it may be molded into any desired form, so that we may have large gears with arms, webs, projecting bosses, counterbalances, etc., to suit the mechanism being designed. These advantages will doubtless continue to keep cast iron the most used of all materials for ordinary work.

In a pair of gears the pinion is often made of steel, from bar stock or a drop forging, even when the larger gear is made from cast iron or some other material. Steel has the advantage over cast iron of being more resilient—that is, it offers a greater resistance to shock

or impact. Since the pinion of a pair of gears naturally has teeth of a weaker form than those of its mate, it should be made of stronger material. Furthermore, there is usually less friction between two different metals in contact than between two parts made of the same material. Still further, the smaller of a pair of gears will wear out much faster than its mate, as each of its teeth is in action a greater number of times in a given period; so on this account as well, it should be made of the more resisting material. As the softer grades of steel, however, are not very durable, steel pinions are sometimes case-hardened, or they may be made of high carbon steel that can be hardened without requiring the action of carbonizing materials. By such processes the pinions gain in durability, but suffer somewhat in accuracy of outline, since the natural result of heat treatment of any kind is to warp and distort the part treated. It is possible to grind hardened gears to the correct outline, leaving a little stock on the sides of the teeth, after cutting, for this purpose. There would, however, seem to be some doubt of the commercial success of the process, owing to the difficulty of keeping the grinding wheel to its proper shape, and keeping the mechanism of the machine itself in proper condition in the presence of the emery dust with which it is surrounded.

Steel blanks for medium sized gears are sometimes drop-forged to bring the wheel to the desired shape. Fairly long hubs, a thickened rim, and a thin web, can be formed in this way without requiring the form to be turned out of solid metal, with the attending waste of time and material. The steel drop forging, thus, has some of the advantages of the casting. It is more costly than the casting, but may be made of better material.

When gears are made of steel from the bar or from forgings, a wide range of physical qualities is offered. The steel may have almost any desired strength, hardness and resilience, or capacity to resist shock. The matter of hardness is important in the case of high-speed gearing. Two soft materials working on each other at a high velocity tend to abrade each other, so one at least of a pair of gears so used should be of a fairly hard grade of steel.

When it is desired to extend to large gears the advantages resulting from the use of steel, making the blanks from round stock or from forgings becomes impracticable, so the use of steel castings is resorted to. This is the best material for large heavy-duty gears. The art of making castings from steel, clean and sound throughout, has only become understood in the past few years. Blow holes, and rough, dirty casting are still common enough, and the use of this material still labors somewhat under the disadvantage of slow deliveries, due to the fact that the steel foundries are not so widely distributed that each machine builder can find one in his own town, as is usually the case with the iron foundry. What has been said as to the qualifications of bar and forged steel as a material for gears, applies also to steel castings, except that there is a narrower range of choice in qualities of the metal, the higher grades not being available for castings.

In general, it may be said that there is a growing tendency to sub-

stitute steel in some form for cast iron as a material for gears. This tendency is especially marked in machine tool design.

It is common and good practice to use a composition metal like brass or bronze for the smaller of a pair of lightly loaded gears which have to run at high speed. When such gears are run with a large gear of cast iron, the difference in texture between the two materials used lessens the friction, and there is a gain on the score of noiselessness as well. Brass may be used where the duty is very light; higher grades of material, like phosphor bronze, are used for heavier service at high speed. When the service becomes quite severe, the materials in the gears should be reversed, so that the larger one is of phosphor bronze, and the smaller one of steel. The pinion has thus its maximum of strength and durability, at the same time that the advantages resulting from the use of dissimilar materials are retained.

Where noiselessness is a prime consideration, rawhide is extensively used. This non-metallic substance possesses the required structure to deaden the sound vibrations, together with a considerable degree of toughness, when properly cured. Manufacturers of gear blanks from this material cure the hide by processes which they claim give far better results for this service than can be obtained by ordinary means. The material is not injured by oil, though it does not require lubrication in service; but there has been some complaint of its swelling and losing its shape when exposed to moisture. Trouble from this source may, however, have been due to the use of an inferior grade of material, because thousands of rawhide pinions are in satisfactory daily use, under all sorts of conditions, at the present time. It is a somewhat more costly material than the others commonly used, but its compensating freedom from noise is often worth more than the added expense. But one of a pair of gears—generally the pinion—is made from this substance, the gear being of steel or iron. Gears as large as 40 inches in diameter have been made from this material.

Fiber is another material used under about the same conditions as rawhide. It is not as strong, and it suffers under the disadvantage of being difficult to machine, owing to its peculiar gritty structure. It is also liable to swell in the presence of moisture. It has an advantage over rawhide in that it is comparatively inexpensive, and may be purchased in a variety of sizes of bars, rods, tubes, etc., so that it is convenient to use at short notice. For light duty at high speed it serves its purpose very well.

Racks of large size, such as those used for driving the platens of metal planers, are made of iron or steel castings. Smaller ones are made from bar steel stock, either machine steel finished all over, or cold rolled steel. The latter material does not require other machining than the cutting of the teeth, being accurately finished to certain convenient sizes in the process of rolling. The cutting of the teeth causes the stock to bend, however, necessitating a straightening operation.

Strength of Gear Teeth

The rule in most common use for determining the strength of gears is the one proposed by Mr. Wilfred Lewis, and described by him in a

paper read before the Engineers' Club of Philadelphia. The utility of this rule is due to its simple form, to the fact that it takes into account a greater number of factors than does any other, and to the fact that the effect of each of these factors is rationally expressed in the formula.

Table VII may be used for finding the allowable unit fiber stress to use in the Lewis formula, for any given speed, for different materials. The values for steel and cast iron are those originally suggested, altered slightly to agree with formula (2), given in the chart on page 29; those for phosphor bronze have been added by the author. It will be noted that two columns of values are given for each material; as

TABLE VII. WORKING STRESSES FOR USE WITH THE LEWIS FORMULA

		Safe Working Unit Stress = S					
		Cast Iron		Phosphor Bronze		Steel	
Velocity in Feet per Minute = V	Strength Factors	Ordinary Workmanship	High-grade Workmanship	Ordinary Workmanship	High-grade Workmanship	Ordinary Workmanship	High-grade Workmanship
0	1.000	6000	8000	9000	12000	15000	20000
100	0.857	5150	6850	7700	10300	12800	17100
200	0.750	4500	6000	6750	9000	11200	15000
300	0.666	4000	5350	6000	8000	10000	13300
450	0.571	3400	4550	5150	6850	8550	11400
600	0.500	3000	4000	4500	6000	7500	10000
900	0.400	2400	3200	3600	4800	6000	8000
1200	0.333	2000	2650	3000	4000	5000	6650
1800	0.250	1500	2000	2250	3000	3750	5000
2400	0.200	1200	1600	1800	2400	3000	4000

explained, the first set of values may be used for workmanship of ordinary grade, while the other is permissible with a higher grade. The second column, of strength factors, may be used for finding the allowable working fiber stress to use for any given speed, when the safe static stress is known; to find the fiber stress, multiply the safe static stress by the strength factor. Formula (2) on page 29 may be used for the same purpose, giving results closely approximating the values of Table VII.

The variable factor introduced into the problem by the varying shape of teeth of the same pitch in gears of different numbers of teeth, is taken care of by introducing in the formula the outline factors Y given in the chart on page 29. These factors are given for that arrangement of the formula which applies to diametral pitches.

The formulas in the chart take no account of any such limitation of the width of face for a gear of given pitch as obtains in practice. The strength is made to increase directly with the width of face, without limit. In practice, if the face is too long in proportion to the size of the tooth, we cannot be sure that each tooth of the gear has a full

bearing over its whole length on its mating tooth in the other gear. The shafts on which the two are mounted may not be parallel, or, if originally parallel, they may deflect under the strain of the load transmitted. For reasons like this, in ordinary commercial work, the width of face may be considered as well proportioned when it equals, in inches, 8.75 divided by the diametral pitch. As a formula, this gives us $A = \frac{8.75}{P}$.

In cases where accurate workmanship can be depended on, there is a gain in using teeth of wider face and finer pitch than would be allowed by the formulas on page 29. There is a gain in efficiency, smoothness of action, and noiselessness, especially at high speeds. In fact, the width of face of a gear may well be made to depend in part on the speed at which it is run, as well as on the pitch, it being taken for granted, of course, that the pitch and width are such as to give the required strength. A suggested relation between the speed and the face is expressed in the following rule, offered by an English engineer as having given good results in an extended practice. It has been changed to derive the answer directly from the diametral pitch, instead of from the circular pitch, as originally given: *To find a well-proportioned width of face for carefully made gearing, multiply the square root of the pitch line velocity in feet per minute by* 0.15, *add* 9 *to the product, and divide the result by the diametral pitch.* As a formula, this gives us:

$$A = \frac{0.15\sqrt{V} + 9}{P}$$

Example.—What should be the pitch and the width of face of a steel pinion, 4 inches in pitch diameter, the teeth shaped according to the 14½-degree involute system, running 750 revolutions per minute, and transmitting 10 horse-power; the workmanship is high grade, and the width of the face is to be proportioned according to the rule and formula given immediately above?

The velocity at the pitch line is $= 0.262\ DR = 0.262 \times 4 \times 750 =$ 786 feet per minute. (See formula (1) in the chart for strength of spur gears.) The allowable running stress for a static stress of 20,000 pounds per square inch is found by formula (2):

$$S = 20{,}000 \times \frac{600}{600 + 786} = 8{,}660 \text{ pounds per square inch.}$$

The load at the pitch line is equal to

$$\frac{10 \times 33{,}000 \times 12}{\pi \times 4 \times 750} = 420 \text{ pounds.}$$

Assume 5 diametral pitch as a trial pitch for the teeth; then the number of teeth equals $5 \times 4 = 20$. Transposing formula (3):

$$W = \frac{S\,A\,Y}{P} \text{ gives us } A = \frac{W\,P}{S\,Y}.$$

List of Reference Letters.

D = pitch diameter of gear in inches.

R = revolutions per minute.

V = velocity in ft. per min. at pitch diameter.

S_s = allowable static unit stress for material.

S = allowable unit stress for material at given velocity.

A = width of face.

Y = outline factor (see table below).

P = diametral pitch (if circular pitch is given, divide 3.1416 by circular pitch to obtain diametral pitch).

C = pitch cone radius.

W = maximum safe tangential load in pounds at pitch diameter.

H.P. = maximum safe horse-power.

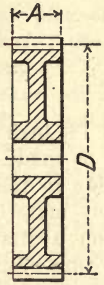


Table of Outline Factors (Y).

Number of Teeth	Outline Factor = Y		Number of Teeth	Outline Factor = Y	
	14½° Involute (Std.) and Cycloidal	20° Involute		14½° Involute (Std.) and Cycloidal	20° Involute
12	0.210	0.245	27	0.314	0.349
13	0.220	0.261	30	0.320	0.358
14	0.226	0.276	34	0.327	0.371
15	0.236	0.289	38	0.336	0.383
16	0.242	0.295	43	0.346	0.396
17	0.251	0.302	50	0.352	0.408
18	0.261	0.308	60	0.358	0.421
19	0.273	0.314	75	0.364	0.434
20	0.283	0.320	100	0.371	0.446
21	0.289	0.327	150	0.377	0.459
23	0.295	0.333	300	0.383	0.471
25	0.305	0.339	Rack	0.390	0.484

Use rules and formulas 1 to 4 in the order given.

No.	To Find	Rule	Formula
1	Velocity in ft. per min. at the pitch diameter	Multiply the product of the diameter in inches and the number of revolutions per minute, by 0.262	$V = 0.262\,DR$
2	Allowable unit stress at given velocity	Multiply the allowable static stress by 600 and divide the result by the velocity in feet per minute plus 600	$S = S_s \times \frac{600}{600+V}$
3	Maximum safe tangential load at pitch diameter	Multiply together the allowable stress for the given velocity, the width of face, the tooth outline factor; divide the result by the diametral pitch	$W = \frac{SAY}{P}$
4	Maximum safe Horse-Power	Multiply the safe load at the pitch line by the velocity in feet per minute, and divide the result by 33,000	$HP = \frac{WV}{33{,}000}$

Apply this transposed formula and find a trial width of face (factor Y is given in the table in the chart):

$$A = \frac{420 \times 5}{8{,}660 \times 0.283} = 0.9 \text{ inch, approximately.}$$

For 5 pitch, however, according to the formula

$$A = \frac{0.15\sqrt{V} + 9}{P}$$

the width of face should be

$$A = \frac{0.15\sqrt{786} + 9}{5} = 2.64 \text{ inch.}$$

Thus, our pitch is evidently too coarse. Repeated trials show that if we make calculations for 9 diametral pitch the results from the two formulas for width of face agree fairly well. Thus:

$$\text{Number of teeth} = 9 \times 4 = 36.$$

$$A = \frac{420 \times 9}{8{,}660 \times 0.332} = 1.32 \text{ inch.}$$

$$A = \frac{0.15\sqrt{786} + 9}{9} = 1.4 \text{ inch.}$$

We may, therefore, settle on 9 diametral pitch and 1⅜-inch width of face, as the dimensions to which the gear ought to be made.

It will be noted that nothing was said about rawhide in the table on page 27, giving the allowable stresses for different materials at different velocities. There is not as much information available as might be desired on the strength of rawhide pinions. One prominent firm, the New Process Raw Hide Co., makes a regular practice of replacing high speed cast iron pinions with those made of rawhide, of the same dimensions. Where, however, a rawhide pinion is to replace a steel gear, working under severe conditions, a special construction is used, in which the weaker material is strengthened by a bronze reinforcement.

Mr. Diefendorf, the chief engineer of the New Process Raw Hide Co., gives the following information on this subject: "Our published statement, that cast iron pinions can be replaced with rawhide ones of the same pitch and number of teeth, holds good down to a peripheral velocity of about 1,600 feet per minute. Figuring on a peripheral velocity of 1,800 feet per minute for a 15-tooth pinion, new process rawhide pinions are good for a load of 150 pounds per inch of face for a 1-inch circular pitch gear, with other pitches in proportion, up to a maximum load not to exceed 250 pounds per inch of face, beyond which we have found it undesirable to go, owing to the compression of rawhide under heavy loads. It would appear that the elastic limit and compression point should be taken into consideration more than the tensile strength, as the material will bend long before it shows any sign of breakage; in fact, it is owing to its elastic qualities that

our rawhide is able to withstand shocks at high speed that would possibly strip the teeth of cast iron pinions."

The standard German engineers' handbook, "Hütte," gives a rule which may be translated into the following form for English measurements: *To find the allowable load in pounds at the pitch line for a rawhide pinion, multiply the width of face in inches by from* 180 *to* 360, *and divide the product by the diametral pitch.* It will be seen that this gives much lower permissible loads than does the New Process Rawhide Co.'s rule, which reduces to a factor of about 470, in place of the 180 to 360 given in "Hütte." In both of these rules the strength is made independent of the velocity at the pitch line. Since decrease of strength with increase of velocity is due to impact, and since rawhide is a substance peculiarly fitted to absorb impact harmlessly, it is logical to assume that the effect of increasing the velocity is negligible. This accounts for the fact that a rawhide gear will be as strong as a cast iron one at high speeds, when it would appear very weak in comparison with it in a static test.

Durability of Gearing

A pair of gears figured by the rules we have just given, to be strong enough for the service they are to undergo, may or may not be so proportioned as to be commercially durable. By "commercially durable" gears, we mean those which will last well in comparison with the rest of the machine of which they are a part. In some classes of machinery, gears strong enough for their work would certainly be commercially durable. A rack and pinion, for instance, used to raise a sluice gate for a dam, if made strong enough, would evidently wear indefinitely, though they might rust away. It is plain that all gearing designed for occasional or intermittent use, even under heavy loads, is strong enough to wear well if it is strong enough to bear the load placed upon it. With gearing used for the continuous transmission of power, however, we cannot be sure of this. The gearing of a drive connecting a motor with a printing press, for instance, might conceivably be strong enough and yet not wear as long as the rest of the machine.

The pinion will naturally wear faster than its mate, since each of its teeth is in action a greater number of times per minute. To make the life of the two more nearly alike, it is customary to make them of different materials, as already mentioned, the pinion being made of the more durable one. Thus, a combination of steel pinion and cast iron gear is common and occasionally conditions are found which warrant the expense of a hardened steel pinion and a phosphor bronze gear. The use of the better material in the smaller gear of the pair is proper from the standpoint of strength as well as from that of durability. An examination of the Lewis outline constants as tabulated in the preceding section of this chapter, will show that the teeth of the pinion are always weaker than those of the gear; so it is necessary, if an excess of strength is to be avoided in the gear, to make the pinion of the stronger material; but if the pinion is a little less durable

than the gear, it will take most of the wear; and being more cheaply renewed than its larger mate, the mechanism is kept up at a less expense. It is not wise to use soft steel in both members for heavy service at high speed.

Where the velocity ratio is not extreme and severe service is exacted, as in automobile gearing, the two members may be made of the same material—hardened, or preferably, case-hardened alloy steel.

Efficiency of Standard Spur Gears

The efficiency of two spur gears (or of any other power transmitting mechanism, for that matter) is measured by the percentage they deliver of the power entrusted to them. Thus, if a water wheel delivers 160 horse-power to the driving pinion on the shaft on which it is mounted, and the mating gear on the jack-shaft transmits 140 horse-power to that shaft, the efficiency of the gearing is $140 \div 160 = 87\frac{1}{2}$ per cent.

To obtain the maximum of efficiency, attention must be paid to the following considerations:

Form the teeth as near to the perfect theoretical shape as good workmanship will bring them, giving the acting surfaces a fine smooth finish. If the teeth are milled to shape, special cutters should be used, made accurately to shape for the exact number of teeth. The gears must be mounted firmly and accurately in their working position.

Use hard, close-grained materials, preferably different for the two gears. Hardened steel on phosphor bronze will probably give the best results, though it is difficult to be sure of the exact shape in hardened gears, unless they are finished by grinding after hardening. Soft steel on soft steel is probably the worst combination so far as efficiency is concerned, though it is stronger than cast iron on cast iron.

Provide continuous and copious lubrication, preferably by an oil or grease bath in an enclosed casing. Lubricated gears should always be enclosed if they are exposed to dust or grit in the slightest degree, otherwise they will grind each other away, and might rather run entirely dry.

Use as fine a pitch as possible, without requiring too wide a face to transmit the power required. The smaller the pitch, the greater the efficiency. Anything that tends to shorten the line of contact, confining it to the vicinity of the pitch point, increases the efficiency, as there is more rubbing at the beginning and end of contact than when the teeth are passing the pitch point.

In general, it may be said that there is no method of transmitting power between two parallel shafts that is more efficient than a pair of well designed and constructed spur gears, working under proper conditions. The highest efficiency, of course, is obtainable only at a considerable expense, so judgment is required to know how far it is wise to carry the possible refinements.

CHAPTER V

DESIGN OF SPUR GEARING

Typical designs of wheels for different materials and uses are shown in Figs. 11 to 16. The one shown in Fig. 11 is the usual one for a steel, cast iron, brass, bronze or fiber pinion. All of its proportions are determined by the gear calculations and the diameter of the shaft on which it is mounted, so there is little to be said about the design. When made from steel, it is generally formed from bar stock in the lathe or screw machine; for other metals, cast blanks are mostly used.

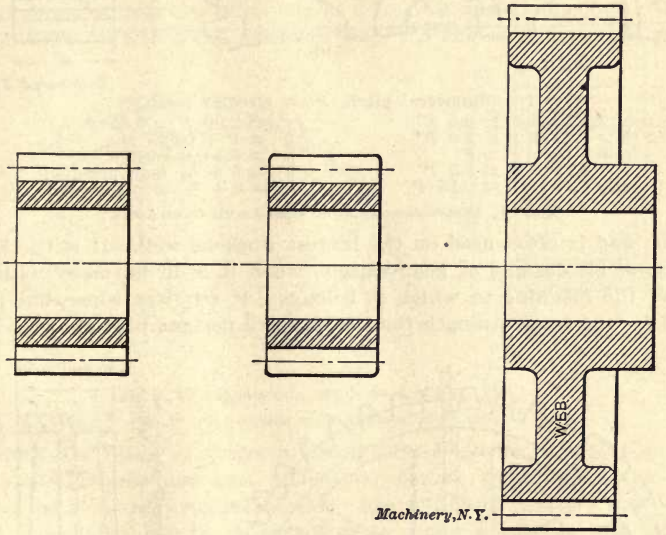
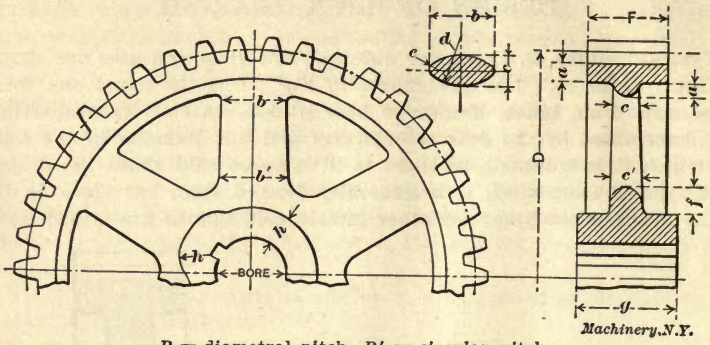


Fig. 11 Fig. 12 Fig. 13

It is the practice of some firms, notably the Brown & Sharpe Mfg. Co. of Providence, R. I., to round the corners of all pinion and gear blanks, large and small, as shown in Fig. 12. This practice has the advantage of making a gear more easy to handle, and less liable to injury in case it is accidentally dropped; it gives it a neater appearance as well.

In Fig. 13 is shown a design used for gears having rather more teeth than those ordinarily known as "pinions." The weight has been lightened by recessing the sides to form the web shown, connecting the rim and the hub. Wheels of this shape are rarely cut from bar stock as the removal of the metal to form the web is too wasteful. The usual practice for this design is to make the blank from a casting or a drop forging.

As the number of teeth for the gear becomes still larger, the increasing weight of the wheel may be lightened by cutting out relieving spaces in the web, or by abandoning the web entirely, and using arms for supporting the rim. This scheme, shown in Fig. 14, with arms of oval section, is the one best adapted for small and medium sized gear

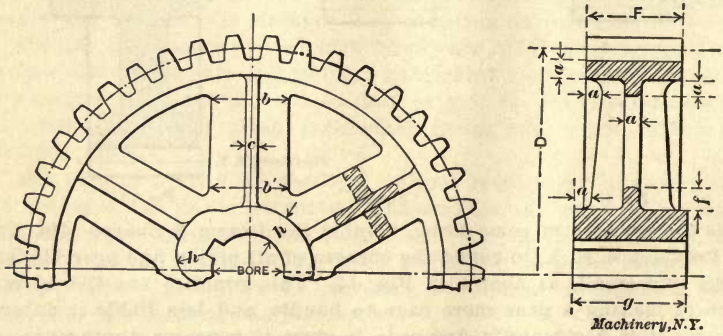


P = diametral pitch, P' = circular pitch.

$a = 1.57 \div P = 0.5\ P'$	$f = 2.00\ P = 0.65\ P'$
$b = 6.28 \div P = 2.0\ P'$	$g = F + 0.025\ D$
$c = 3.14 \div P = P'$	$h = 0.44 \times$ bore
$d = 4.71 \div P = 1.5\ P'$	$b' = b +$ ¾ inch per foot
$e = 0.79 \div P = 0.25\ P'$	$c' = c +$ ¾ inch per foot

Fig. 14. Dimensions of Spur Gears with Oval Arms

blanks, and is often used on the largest work as well. It is the handsomest of all designs of gear wheels, when it is in harmony with the rest of the machine to which it belongs. It requires somewhat more metal for the same strength than do the two designs next shown. It is



P = diametral pitch, P' = circular pitch.

$a = 1.57 \div P = 0.5\ P'$	$g = F + 0.025\ D$
$b = 7.85 \div P = 2.5\ P'$	$h = 0.44 \times$ bore
$c = 0.94 \div P = 0.3\ P'$	$b' = b +$ ¾ inch per foot
$f = 2.20 \div P = 0.7\ P'$	

Fig. 15. Dimensions of Spur Gears with Ribbed Arms

very easily molded. Suitable dimensions for wheels of various sizes made in this way, are tabulated below the illustration.

For the largest gears, made of steel, cast iron or bronze castings, wheels with arms of + or H-section are largely used. Dimensions

for wheels of these types are given in Figs. 15 and 16. In these designs, the metal is so distributed as to give a high degree of rigidity for the weight. These two forms, particularly that in Fig. 16, are more difficult to mold than those previously shown. The latter form is better for gears whose faces are very wide in proportion to their pitch, than either of the two in Figs. 14 and 15.

The tabular dimensions given for the various forms of wheels are to be considered as suggestive rather than authoritative. The tables have been in constant use for some years, however, and have proved to be very satisfactory. "Draft," for removing the patterns from the sand in molding, is not shown in any of the illustrations. It should be provided liberally, and should be added to the dimensions given, rather than taken off.

The Governing Conditions in the Design of Gearing

The problem of gear design is one of materials and dimensions. The considerations on which the designer bases his choice of materials and

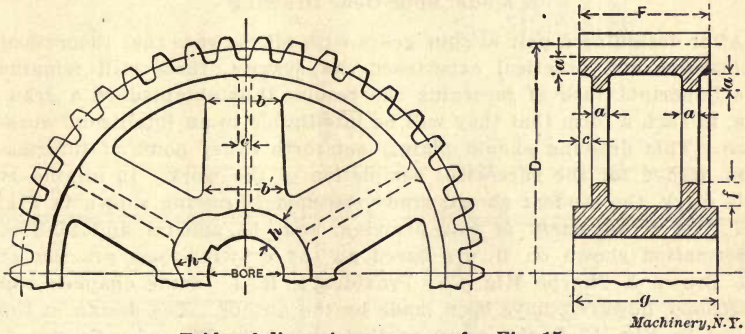


Table of dimensions same as for Fig. 15.

Fig. 16. Spur Gear with Arms of H-section

dimensions are those of strength, durability, efficiency, smoothness of action, noiselessness and cost. The gear cannot attain perfection in all these particulars, as some of them are mutually hostile; the item of cost, especially, has to be sacrificed to make a gain in any other direction. The problem of design is thus one of compromise, and the designer has only his judgment to rely on in determining the relative importance of the various considerations.

It is possible, however, to lay down a few simple rules along this line. The prime consideration is that of strength. If the teeth of the gear are not strong enough to transmit the pressure they are calculated on to bear, the gear will break, and the other virtues it may possess in the way of cheapness, noiselessness, etc., will be of no avail. As has already been stated, in gearing subjected to occasional use only, the durability is sufficient for all practical purposes if the strength is sufficient; but there is a possibility that gearing transmitting power at high speed may wear out before it breaks. Where gearing is used, as in automatic machinery, primarily to obtain certain desired movements in the mechanism, without requiring the transmission of any

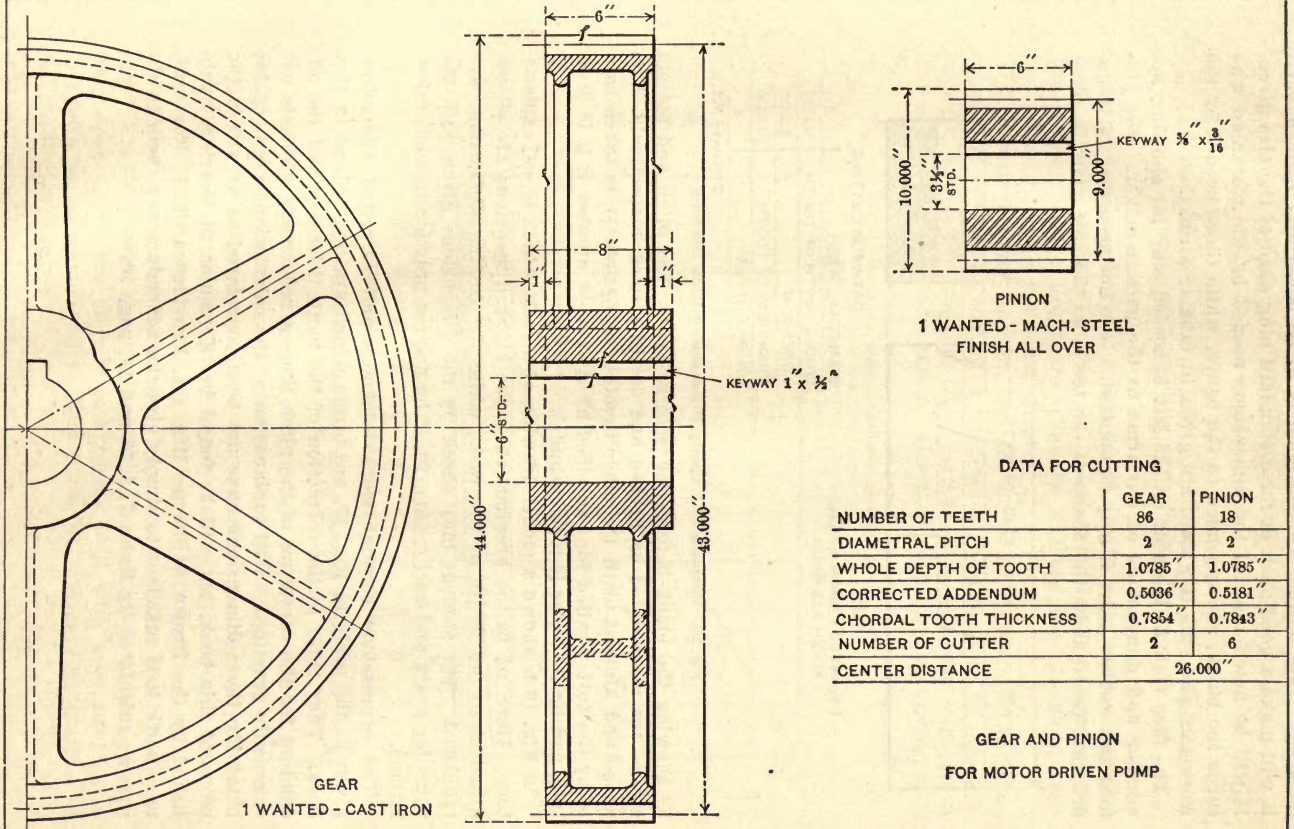
great amount of power, the question of efficiency is not of prime importance. On the other hand, when the main object of a pair of gears is to receive so many horse-power from one shaft, and transmit as nearly as possible the same amount to another shaft, the question of efficiency becomes one worthy of the most careful consideration. As to smoothness of action, if the requirements for efficiency and silence have been met the gears will run smoothly, without shock or vibration. Noiselessness, as a problem in design, is largely a matter of the selection of materials, always supposing that the teeth of the gears are formed to correct tooth curves. The matter of cost is in part an engineering problem, and in part a commercial one. It is an engineering problem in so far as it bears on the problem of obtaining the greatest perfection in the other particulars enumerated, with a given expenditure; and it is a commercial problem when it comes to determining how great a degree of refinement it is advisable to ask the purchaser of the finished machine to pay for.

A Model Spur Gear Drawing

After designing a pair of spur gears with all the care that theoretical knowledge and practical experience can suggest, there still remains the important task of recording the results thus obtained on a drawing, in such a form that they will be intelligible to an intelligent workman. This drawing should plainly set forth every point of information needed for the successful completion of the work. In aiming at this mark, the student should study the model drawing shown in Fig. 17. The arrangement of this drawing, and the amount and kind of information shown on it, are based on the drawing-room practice of the Brown & Sharpe Mfg. Co., Providence, R. I. Some changes and additions, however, have been made by the author. The design of the wheel in Fig. 17 is the same as that shown in Fig. 16. As stated, this design is not so easily molded as that in Fig. 15, but it is the most suitable form for gears of a comparatively wide face. No pattern dimensions are given. A drawing for machine shop use should not be confused by a maze of dimensions which are not used by the machinist. The patternmaker can be taken care of by a separate drawing, or by a special blue-print with his dimensions put on in yellow pencil.

The dimensions given are, perhaps, figured somewhat closer than is required on work of this size. The important dimensions, such as the outside diameter and the center distance, on which depend the proper fitting of the teeth of the gear, are a little too large to be measured with the vernier caliper, but they should surely be accurate within 0.005 inch—a limit easily attainable by a skillful workman.

Where a definite amount of allowable variation from the exact size can be determined on, it is customary in interchangeable manufacturing to give the dimensions with maximum and minimum limits. In work of the kind shown in Fig. 17, however, where the machine is "built" rather than "manufactured," it is not usual to give limits. The diameter of the hole in the hub of the gear is given as "6" Std." on the



DATA FOR CUTTING

	GEAR	PINION
NUMBER OF TEETH	86	18
DIAMETRAL PITCH	2	2
WHOLE DEPTH OF TOOTH	1.0785″	1.0785″
CORRECTED ADDENDUM	0.5036″	0.5181″
CHORDAL TOOTH THICKNESS	0.7854″	0.7843″
NUMBER OF CUTTER	2	6
CENTER DISTANCE	26.000″	

GEAR AND PINION

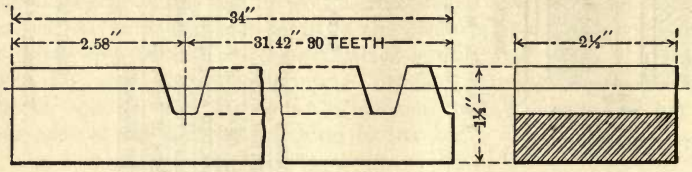
FOR MOTOR DRIVEN PUMP

Machinery, N.Y.

Fig. 17. Example of Properly Dimensioned Gear Drawing

drawing. This means that the hole is to be bored and reamed until it will make a good push fit for a standard plug gage of the size given. It will be noted that all the dimensions needed by the workman who turns the blank, are appended to the figure, while those needed by the workman who cuts the teeth are given in tabular form.

The face view of the gear on the left is needed only for showing the number and dimensions of the arms to the pattern-maker. For pinions and webbed gears it may be omitted. It is not necessary in standard gearing to show the shape of the teeth, so the side view is given



RACK
1 WANTED - MACH. STEEL
FINISH ALL OVER

DATA FOR CUTTING

DIAMETRAL PITCH	3
WHOLE DEPTH OF TOOTH	0.7190″
ADDENDUM	0.3333″
THICKNESS OF TOOTH	0.5236″
NO. OF CUTTER	1

Machinery, N.Y.

Fig. 18. Example of Properly Dimensioned Rack Drawing

as showing the blank before the teeth are cut. The pitch and bottom circles are represented by broken and dotted circles, respectively. The shape and kind of teeth (whether involute or cycloidal) is taken care of by the cutter called for—specified by its proper number if it is involute, and by its letter if it is cycloidal.

In Fig. 18 is shown a model drawing of a rack, which is self-explanatory. Here, as in the previous case, the blank dimensions are shown attached to the figure of the rack, while the cutting dimensions are tabulated. The student may check up the dimensions given with the rules for gears and racks, page 20, if he desires practice in such calculations.

The expressions chordal tooth thickness and corrected addendum given in the table in Fig. 17, are terms which are not defined in this book. They refer to the correction of the tooth thickness and the addendum for the curvature of the pitch line—a refinement which is not commonly practiced. Full explanation of the calculations required for obtaining these dimensions have not been considered to be within the scope of this book, but will be found in "Formulas in Gearing," published by the Brown & Sharpe Mfg. Co., Providence, R. I. For ordinary work it is sufficient to simply give the addendum and tooth thickness as calculated by Rules 7, 8, 13 and 14, page 20.

CHAPTER VI

VARIATION OF THE STRENGTH OF GEAR TEETH WITH THE VELOCITY*

The generally accepted formula for calculating the strength of gear teeth is that proposed by Mr. Wilfred Lewis, first published in the Proceedings of the Engineers' Club of Philadelphia, January, 1893, and referred to in a preceding chapter.

The merit of this formula lies in the great number of variables taken into account as compared with other rules in more or less common use, and in the fact that these variables are rationally considered. The effect of each of them can be calculated with some assurance, with

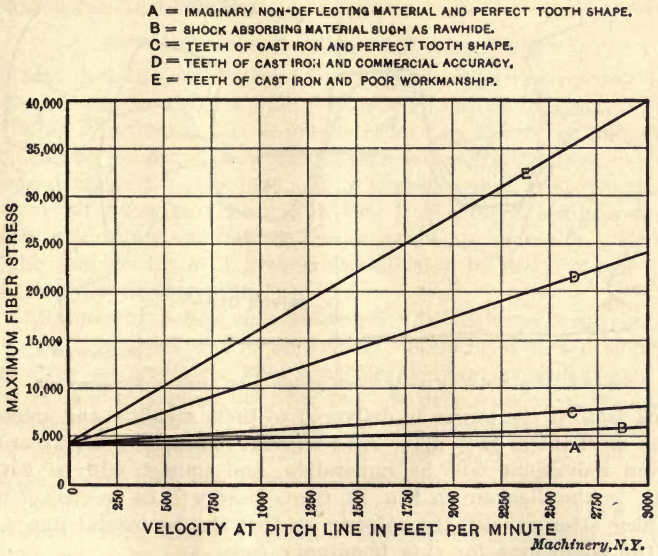


Fig. 19. Hypothetical Diagram showing the Relation of the Velocity to the Fiber Stress

the single exception of the influence of the velocity on the safe stress. In the fifteen years since the formula was first proposed, the original values for the stress as affected by the velocity have been largely used. Many designers, however, have felt that these values are rather unsatisfactory, although most of them will agree that they err rather on the side of safety than otherwise. By referring to Mr. Lewis' original paper it will be seen that these values were not given as being definitely determined, but merely as agreeing well with successful cases met with in his own practice. The following is a general analysis of the conditions involved.

* MACHINERY, January, 1908.

A variation in the strength of the teeth of a gear, due to a variation in the velocity, can be due, of course, to but one thing—impact. To illustrate this idea, and to show the cause of the impact, we will study the action of gearing under three different conditions.

1. *Gears of an imaginary undeflectable material.*—In Fig. 19 is a diagram in which the horizontal distances give velocity in feet per minute, and vertical distances give stresses in pounds per square inch, starting in this case at 4,000, which is assumed to be the maximum fiber stress in the gear we are considering, due to the load at the pitch line, which is supposed to be constant at all speeds. If the teeth of this gear are perfectly formed and well fitted together, so that there is

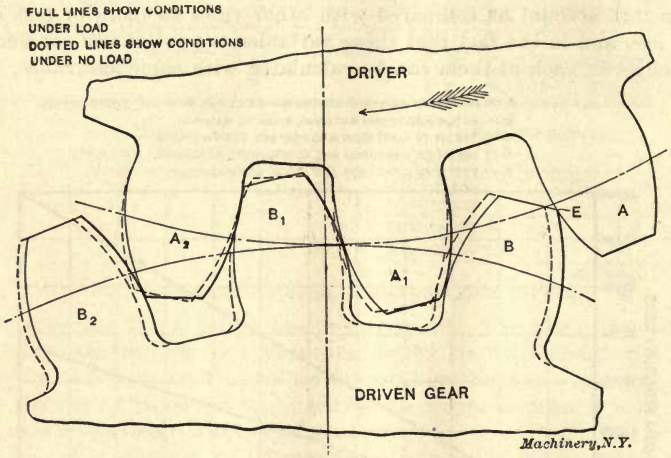


Fig. 20. The Action of Gear Teeth under Load, Greatly Exaggerated

no back lash, if the power is delivered to them steadily and smoothly, and the mechanism they drive runs without shock, any disturbance of the even movement will be impossible, and impact will be entirely absent. In the diagram in Fig. 19, then, there will be no rise of maximum fiber stresses with the velocity, so that the horizontal line *A* will show the conditions for this imaginary case.

2. *With commercial material and theoretically accurate workmanship.* The conditions in this case are shown in Fig. 20, with all the phenomena greatly exaggerated. The full lines show the conditions under load, while the dotted outlines show the conditions when the load is removed from the driven gear. The teeth A_1, B_1, and A_2, B_2, carrying the load, are deflected by it, as shown. Tooth *B*, just about to come into contact with tooth *A*, is on that account shifted from its normal position; it should be located as shown by the dotted lines. If it were in this position, it would come in contact with tooth *A* under mathematically perfect conditions, and there would be no shock of engagement. As it is, the two come suddenly into action as shown at *E*, under different conditions than those contemplated by the design, thus the contact takes place in the form of a slight blow, after which

the teeth are deflected more and more, until they have taken up their share of the load, as shown later at A_1 and B_1. If the gears are moving very slowly, the deflection takes place very slowly, and the problem is practically a static one. If the gears are running at a high velocity, the problem becomes essentially a dynamic one, and the stresses are greater than with the slow speed. The increase in stress with the increase in speed for this second case could probably be represented by a line something like *C*, in Fig. 19.

3. *With commercial materials and commercial accuracy.* This is, of course, the practical case to consider. A line to show the relation of the velocity to the maximum fiber stress for a given gear, would very probably look something like *D* in Fig. 19. This is, in fact, approximately the line which embodies the conclusions of the Lewis tables for a static stress of 4,000 pounds. It is considerably higher than line *C*, because impact due to irregular tooth outlines is added to the impact due to the deflection.

Practical Considerations Affecting Design

The fact that the variation of the strength with the velocity is due to impact, suggests also a number of points relating to design.

1. *Value of accuracy.* It is evident that this theory of impact puts a premium on accuracy in workmanship for gears that are to run at high speed under a heavy load. It is probable that the strength of a given pair of gears may be cut in two if the tooth outlines are not carefully determined, and if the cutter is not set centrally. This suggests the desirability of a greater sub-division of the standard cutter series for work of this kind.

2. *Resilience of design and materials.* In high-speed gearing it is evident that the shock due to the impact should be absorbed as quickly and as fully as possible. This suggests the use, at abnormally high speeds, of rawhide, wood, etc., for one of the members of the pair of gears. The introduction of spring couplings or similar devices may also be desirable, especially where the other parts of the mechanism are liable to transmit shock to the gearing.

3. *Easing off the points of the tooth.* There has always been a sort of superstition that the points of the tooth should be eased off to make the action smoother. This is done, of course, in standard involute gears, though for another reason, that of avoiding interference with the flanks of the pinions. It can now be seen that there is a solid basis for this practice in all cases where gears are to run at such speeds that severe impact is liable to take place. Referring to Fig. 20, teeth *A* and *B* are taking up the load very suddenly, owing to the fact that they are out of step, due to the deflection of the other teeth momentarily carrying the load. Easing away the points of *A* and *B* would mitigate this sudden reception of the load, allowing the inevitable deflection to take place more slowly, with a consequent gain in the strength of the gear at high speeds.

CHAPTER VII

SIMPLIFIED FORMULAS FOR STRENGTH OF GEARS*

It is generally conceded that the Lewis formula for the strength of gear teeth, with its accompanying tables, is the most accurate in form, as the maximum strength of each tooth is determined from its shape. It may be safely used for determining the strength of gears made by modern methods, but its tabulated form makes it difficult to use from the standpoint of the designer. It is well adapted to determine the strength of any given gear or pinion. But the reverse process—that of finding a gear suitable to meet the condition of a given horse-power and revolutions per minute is not so simple, the trial-and-error method being a lengthy one at the best. The following deductions give close and rapid approximations for preliminary work.

As both the gear and its pinion are usually made of the same material, either cast iron or cast steel, the strength of the pair is determined by the strength of its weakest member, which is the pinion when made of the same metal as the gear. For economical reasons the pinion is usually limited to about 15 teeth, so we may take that number as a convenient base. Circular pitch is used in the calculations, but the circular pitch can finally be transformed to diametral pitch if this is desired.

In a train of gears, the maximum reduction on any pair is usually taken at 4 or 5 to 1, so the number of reductions and ratios may be quickly deduced. Then the problem is usually presented as follows:

Given the horse-power and revolutions per minute of the pinion, what will be the allowable working stress, pitch, face, factor of strength and diameter?

The majority of trade gear lists give the horse-power of gears at 100 r. p. m. with an allowable stress for cast iron of 3,000 pounds per square inch. But it is more difficult to transform this horse-power to suit the other conditions, than to proceed independently.

The values of S, the safe working stress, which Mr. Lewis adopted tentatively, as they gave satisfactory results in practice, were as follows, (See also Table VII):

Let V = speed of teeth in feet per minute and S = safe working stress, then

For V =	100 (or less)	200	300	600	900	1,200	1,800	2,400
For cast iron:								
S =	8,000	6,000	4,800	4,000	3,000	2,400	2,000	1,700
For cast steel:								
S =	20,000	15,000	12,000	10,000	7,500	6,000	5,000	4,300

When these values are plotted, it will be seen that the curves, though slightly irregular, closely approximate curves of the hyperbolic form.

* MACHINERY, August, 1909.

The equations of the curves which most nearly agree with the Lewis values, are found to be the following:

$$\text{For cast iron, } S = \frac{88{,}000}{\sqrt{V}}.$$

$$\text{For cast steel, } S = \frac{220{,}000}{\sqrt{V}}.$$

These formulas give the following comparative values:

When $V =$	100	200	300	600	900	1,200	1,800	2,400
For cast iron:								
$S =$	8,800	6,250	5,000	3,600	2,930	2,540	2,080	1,790
For cast steel:								
$S =$	22,000	15,625	12,500	9,000	7,325	6,350	5,200	4,475

The agreement with the Lewis assumed values is remarkably close. The new values will probably come much nearer the true ones, as they are in much better line. They are also much more dependable, as the stress suitable for any speed can be easily found from the formula to the fraction of a pound, if desired, on a true curve; whereas, the use of the tabular values results in the substitution of values which descend by variable steps of from 2,000 to 300 pounds at a jump, or if ordinary interpolation is used the result is still inaccurate, as the interpolation necessarily follows a straight line between the two nearest values, and is thus too high. The new curve values also come nearer to the comparative Harkness values as given by Kent.

The face of gears, A, is another variable quantity; but in the manufacturer's standard lists of to-day the face is usually about 3 times the pitch, and this may be adopted as close enough for preliminary work. It will be found that the majority of stock gears have either 15-degree involute or cycloidal teeth, so these styles will be used in these calculations. The factor of strength, Y', in the Lewis tables for a 15-tooth pinion of these types is 0.075. The factor Y' is found from the values of factor Y in the chart on page 29, by dividing the value of Y, as given, by 3.14. We have, therefore, the following data for a 15-tooth cast iron spur pinion:

Let S = safe working stress, in pounds,
P' = circular pitch, in inches,
A = face, in inches,
Y' = factor of strength,
V = speed of pitch line, in feet per minute.

The Lewis general formula reduces to

$$\text{H. P.} = \frac{SP'AY'V}{33{,}000}$$

From our average determination above, we have:

$$S = \frac{88{,}000}{\sqrt{V}};\ A = 3P';\ Y' = 0.075.$$

Substituting these values in the general formula and reducing, we have for a 15-tooth cast iron spur pinion:

$$\text{H. P.} = 0.6\, P'^2 \sqrt{V} \quad \ldots\ldots (1)$$

By a similar process, we find for a 15-tooth cast steel spur pinion:

$$\text{H. P.} = 1.5\, P'^2 \sqrt{V} \quad \ldots\ldots (2)$$

For a bevel pinion, let

d = small diameter of bevel,

D = large diameter of bevel.

$$\text{Then H. P.} = \frac{SP'AY'V}{33{,}000} \times \frac{d}{D}.$$

As $\frac{d}{D}$ usually equals about $\frac{2}{3}$, we can say:

$$\text{H. P.} = \frac{SP'AY'V}{33{,}000} \times \frac{2}{3}$$

and for a 15-tooth cast iron bevel pinion,

$$\text{H. P.} = 0.4\, P'^2 \sqrt{V} \quad \ldots\ldots (3)$$

For a 15-tooth cast steel bevel pinion,

$$\text{H. P.} = P'^2 \sqrt{V} \quad \ldots\ldots (4)$$

We now wish to find V in terms of revolutions per minute. For a 15-tooth pinion, approximately:

$$V = \frac{15 \times \text{r.p.m.} \times P'}{12} = 1.25 \text{ r.p.m.} \times P'.$$

Substituting this value in (1) we have:

$$\text{H. P.} = 0.6\, P'^2 \sqrt{1.25 \text{ r.p.m.} \times P'}.$$

Squaring, $\text{H. P.}^2 = 0.36\, P'^4 (1.25 \text{ r.p.m.} \times P')$.

Reducing, and solving for P', we have for cast iron spur pinion:

$$P' = \sqrt[5]{\frac{2.22 \text{H.P.}^2}{\text{r.p.m.}}} \quad \ldots\ldots (5)$$

A similar substitution and reduction in formulas (2), (3) and (4) gives the following:

For cast steel spur, $P' = \sqrt[5]{\dfrac{0.36 \text{ H. P.}^2}{\text{r.p.m.}}}$ ..... (6)

For cast iron bevel, $P' = \sqrt[5]{\dfrac{5.0 \text{ H. P.}^2}{\text{r.p.m.}}}$ ..... (7)

For cast steel bevel, $P' = \sqrt[5]{\dfrac{0.8 \text{ H. P.}^2}{\text{r.p.m.}}}$ ..... (8)

For rapidly varying loads, or where there is much starting and stopping, it is well to reduce the safe stress to two-thirds that allowed by the above formulas. We then have:

For cast iron spur, H.P. $= 0.4\sqrt{P'^2}\,V$; $P' = \sqrt[5]{\dfrac{5\text{ H. P.}^2}{\text{r.p.m.}}}$ ... (9)

For cast steel spur, H.P. $= P'^2\sqrt{V}$; $P' = \sqrt[5]{\dfrac{0.8\text{ H.P.}^2}{\text{r.p.m.}}}$ ..... (10)

For cast iron bevel, H.P. $= 0.27\ P'^2\sqrt{V}$; $P' = \sqrt[5]{\dfrac{11.0\text{ H.P.}^2}{\text{r.p.m.}}}$ (11)

For cast steel bevel, H.P. $= 0.67\ P'^2\sqrt{V}$; $P' = \sqrt[5]{\dfrac{1.8\text{ H. P.}^2}{\text{r.p.m.}}}$ (12)

The fifth root can be easily determined by logarithms on the slide rule, or from the usual tables, but the values for the common cases are given later.

Corrections for Tooth Numbers

It now remains to determine the correction for different numbers of teeth.

As the teeth of pinions generally range from 12 to 30, we need not go outside these limits. Let $N =$ number of teeth. Plotting the Lewis values for Y' for this case, and determining the nearest curve, we find that the straight line formula:

$$Y' = \frac{2N + 45}{1{,}000}$$

expresses this curve very closely, as will be seen by the following comparative table:

No. of Teeth, N	Y' by Formula	Y' from Lewis' Tables	No. of Teeth, N	Y' by Formula	Y' from Lewis' Tables
12	0.069	0.067	19	0.083	0.087
13	0.071	0.070	20	0.085	0.090
14	0.073	0.072	21	0.087	0.092
15	0.075	0.075	23	0.091	0.094
16	0.077	0.077	25	0.095	0.097
17	0.079	0.080	27	0.099	0.100
18	0.081	0.083	30	0.105	0.102

Therefore, for other teeth, we can multiply the horse-power given in the above formulas by $\dfrac{2N + 45}{75}$, or more briefly by $0.027\,N + 0.6$.

Correction for Increased Velocity

We must also correct for the increased velocity of this larger pinion, *i. e.*, multiply the result by $\sqrt{\dfrac{N}{15}}$ or $0.26\sqrt{N}$. The continued product of these last two multipliers might be used, but this does not simplify the calculation. These corrections need seldom be applied for preliminary work.

To Find the Pinion Diameter

Lastly, to find the diameter of the pinion, approximately:

$$\text{diameter} = \frac{N \times P'}{\pi}, \text{ or}$$

$$\text{diameter} = 0.318\, N\, P',$$

or for a 15-tooth pinion,

$$\text{diameter} = 4.77\, P' \quad \ldots\ldots (13)$$

If diametral pitch is desired, it is sufficiently close to say:

$$\text{diametral pitch} = \frac{3}{P'} \quad \ldots\ldots (14)$$

The following formulas, therefore, Nos. (5) to (14) (as deduced above), give closely enough for all preliminary determinations, the size of pinion required of 15 teeth.

	Lewis' Tables	Stress ⅔ Lewis' Tables
Cast iron spur, $P' =$	$\sqrt[5]{\frac{2.22\ \text{H.P.}^2}{\text{r.p.m.}}}$	$\sqrt[5]{\frac{5.0\ \text{H.P.}^2}{\text{r.p.m.}}}$
Cast steel spur, $P' =$	$\sqrt[5]{\frac{0.36\ \text{H.P.}^2}{\text{r.p.m.}}}$	$\sqrt[5]{\frac{0.8\ \text{H.P.}^2}{\text{r.p.m.}}}$
Cast iron bevel, $P' =$	$\sqrt[5]{\frac{5.0\ \text{H.P.}^2}{\text{r.p.m.}}}$	$\sqrt[5]{\frac{11.0\ \text{H.P.}^2}{\text{r.p.m.}}}$
Cast steel bevel, $P' =$	$\sqrt[5]{\frac{0.8\ \text{H.P.}^2}{\text{r.p.m.}}}$	$\sqrt[5]{\frac{1.8\ \text{H.P.}^2}{\text{r.p.m.}}}$

$$\text{Diameter} = 4.77\, P'.$$

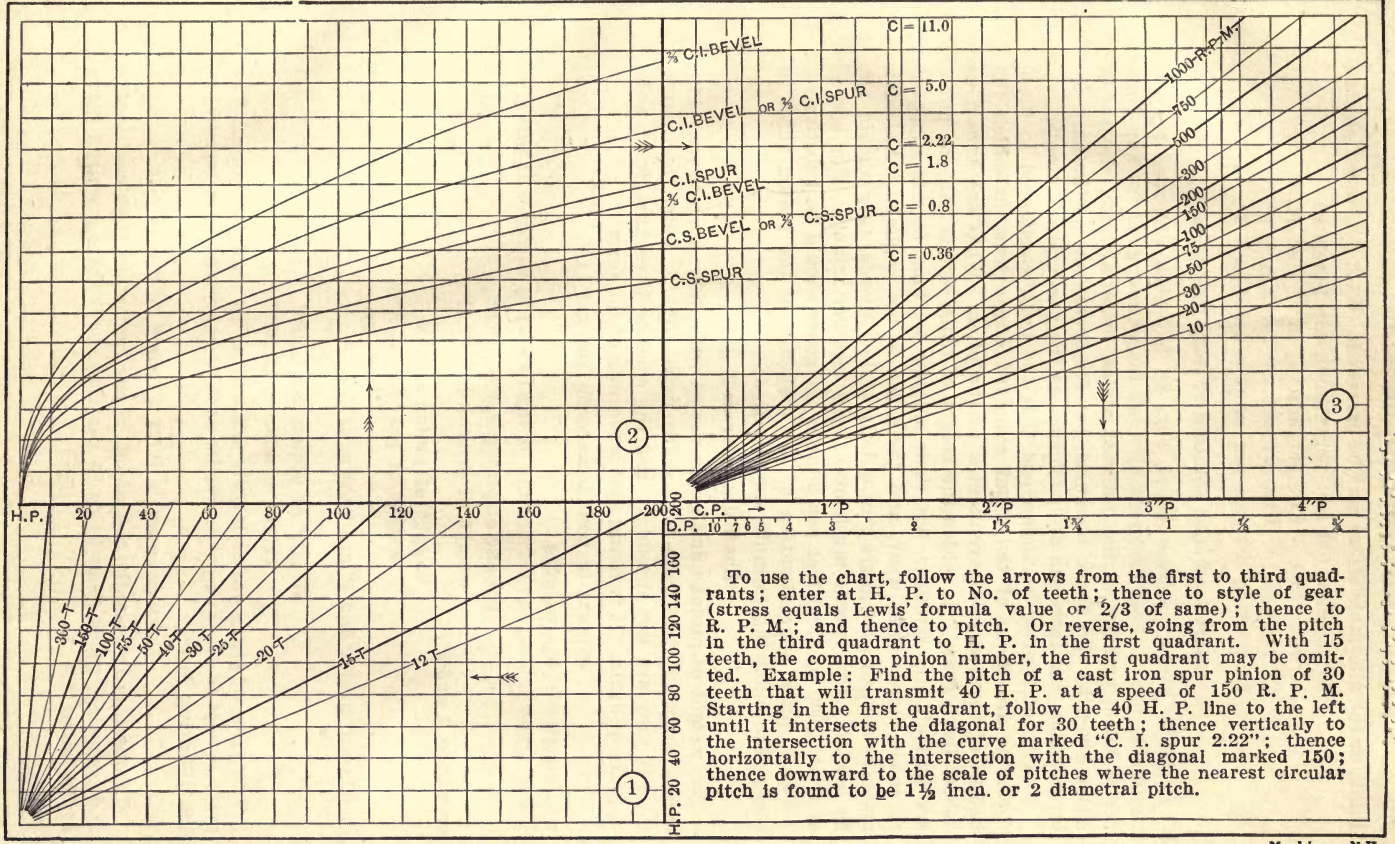
$$\text{Diametral pitch} = \frac{3}{P'}.$$

Practically, stock gears are made up to 3 inches circular pitch by ¼-inch steps, and a pitch of less than 1 inch is seldom used.

The following table will therefore determine the roots for the nearest common pitch:

No. or Root	Fifth Power	No. or Root	Fifth Power	No. or Root	Fifth Power
¾	0.24	2	32	3½	525
1	1	2¼	58	4	1,024
1¼	3	2½	98	4½	1,845
1½	8	2¾	158	5	3,125
1¾	16	3	243	6	7,776

In case the revolutions per minute of the pinion are less than 80, which is exceptionally slow, care must be taken in applying the for-



To use the chart, follow the arrows from the first to third quadrants; enter at H. P. to No. of teeth; thence to style of gear (stress equals Lewis' formula value or 2/3 of same); thence to R. P. M.; and thence to pitch. Or reverse, going from the pitch in the third quadrant to H. P. in the first quadrant. With 15 teeth, the common pinion number, the first quadrant may be omitted. Example: Find the pitch of a cast iron spur pinion of 30 teeth that will transmit 40 H. P. at a speed of 150 R. P. M. Starting in the first quadrant, follow the 40 H. P. line to the left until it intersects the diagonal for 30 teeth; thence vertically to the intersection with the curve marked "C. I. spur 2.22"; thence horizontally to the intersection with the diagonal marked 150; thence downward to the scale of pitches where the nearest circular pitch is found to be 1½ inch, or 2 diametral pitch.

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Fig. 21. Three Quadrant Gear Chart for Solution of Spur and Bevel Gear Problems, based on Lewis' Formula; for 15-degree Involute and Cycloidal Teeth

mula, or the allowable stress may be exceeded. With a 15-tooth pinion:

80 r.p.m. = 100 feet per minute for 1-inch P'.
40 r.p.m = 100 feet per minute for 2-inch P'.
27 r.p.m = 100 feet per minute for 3-inch P'.
20 r.p.m = 100 feet per minute for 4-inch P'.

Chart for Rapid Solution of Gear Problems

A simple three quadrant chart, Fig. 21, has been prepared for the rapid solution of these problems by mere inspection, good for any number of teeth, and for all the different styles, materials, and stresses of gears given by the above formulas, but for occasional preliminary determination, the formulas are sufficient, as their solution is simple.

It will, of course, be understood that the teeth considered in these formulas are those of the usual standard dimensions, in which the height of tooth equals seven-tenths of the pitch. What are known as "short tooth gears," in which the height of tooth equals half the pitch, are undoubtedly stronger, but their smaller working face is supposed to cause more rapid wear, and their use is not common. Although machine-molded cast gears run quietly at low speeds, they should not be used for rim speeds much over 1,000 feet per minute. For speeds of from 1,000 to 3,000 feet per minute cut gears should be substituted.

For a quick approximation of the diameter of the pinion shaft in inches the following formula may be used:

$$\text{Shaft diameter} = P' + 1.$$

The weight of pinions and gears varies with different makers. Pinions of from 12 to 30 teeth are usually made slightly wider than gears, even if they are not shrouded; and the smaller sizes have solid webs in place of arms. It is found that a formula of the form

$$\text{Weight in pounds} = \text{coefficient} \times P'^2AN,$$

will usually fit the weights.

For many tables, the coefficients of the following values will serve:

$$\text{Weight of pinion} = 0.35\,P'^2AN,$$
$$\text{weight of gear} = 0.45\,P'^2AN,$$

or where $A = 3\,P'$,

$$\text{Weight of pinion} = P'^3N,$$
$$\text{weight of gear} = 1.35\,P'^3N$$

or when diameter and P' are known, as $N = \frac{\pi D}{P'}$,

$$\text{Weight of pinion} = 3.1\,DP'^2,$$
$$\text{weight of gear} = 4.2\,DP'^2.$$

The price of gears varies largely with different manufacturers. The price of cast tooth spur gears can be usually expressed by a formula of the following form:

$$\text{Price} = (\text{coeff.} \times P'N) + (\text{coeff.} \times P').$$

Cut tooth gears usually cost about 20 per cent more than cast tooth; and cast steel gears from 50 to 75 per cent more than cast iron gears of the same size.

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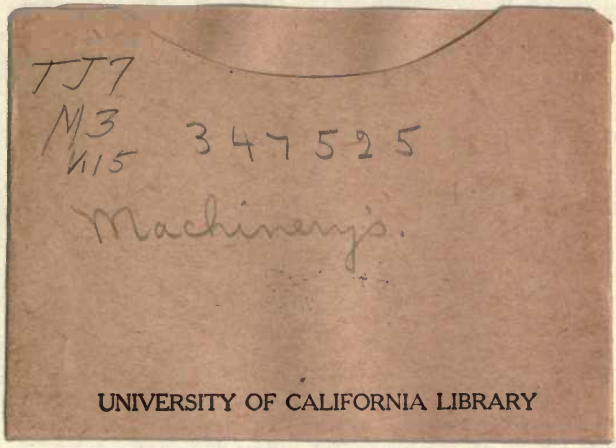
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